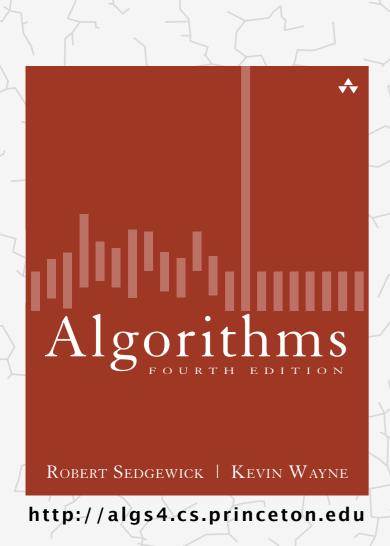
Algorithms



PRIM'S ALGORITHM DEMO

- Prim's algorithm
- lazy implementation
- eager implementation

PRIM'S ALGORITHM DEMO

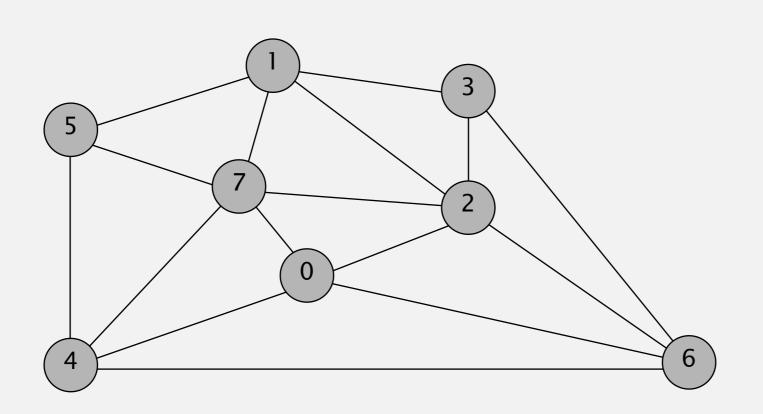
- Prim's algorithm
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Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

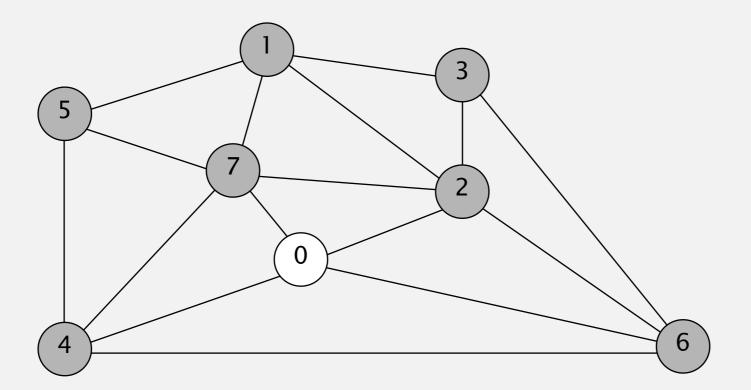
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



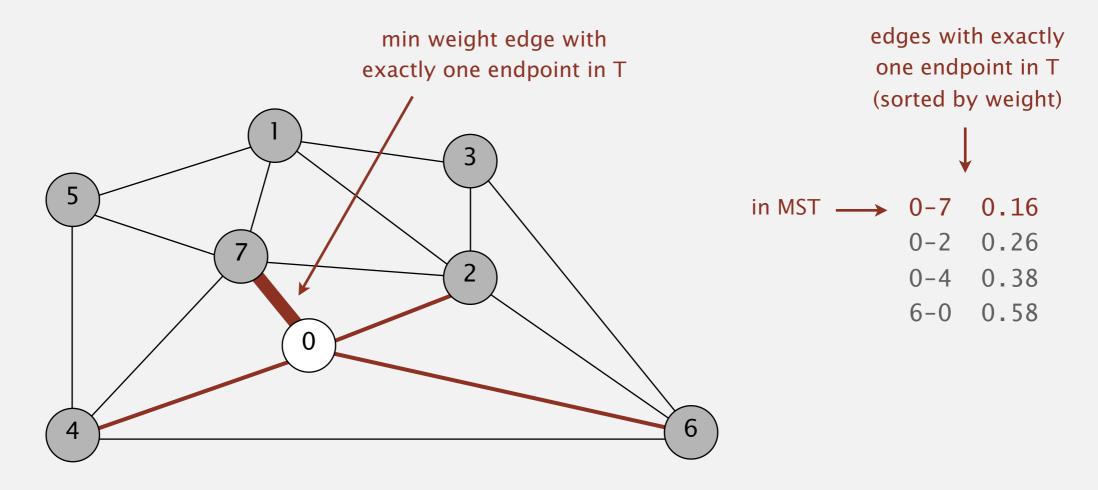
an edge-weighted graph

| 0-7 | 0.16 |
|-----|------|
| 2-3 | 0.17 |
| 1-7 | 0.19 |
| 0-2 | 0.26 |
| 5-7 | 0.28 |
| 1-3 | 0.29 |
| 1-5 | 0.32 |
| 2-7 | 0.34 |
| 4-5 | 0.35 |
| 1-2 | 0.36 |
| 4-7 | 0.37 |
| 0-4 | 0.38 |
| 6-2 | 0.40 |
| 3-6 | 0.52 |
| 6-0 | 0.58 |
| 6-4 | 0.93 |

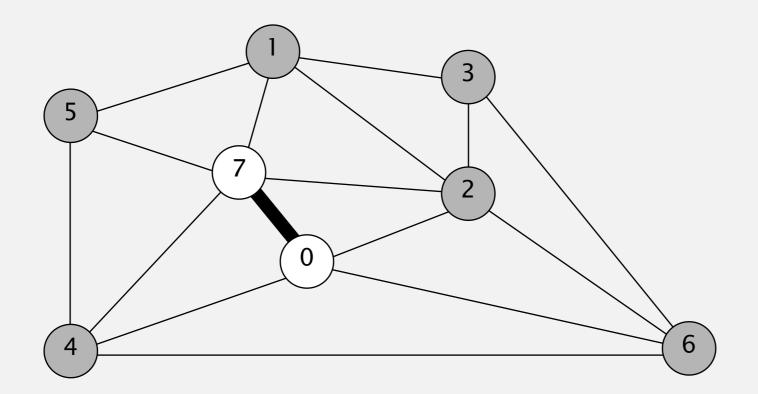
- Start with vertex 0 and greedily grow tree *T*.
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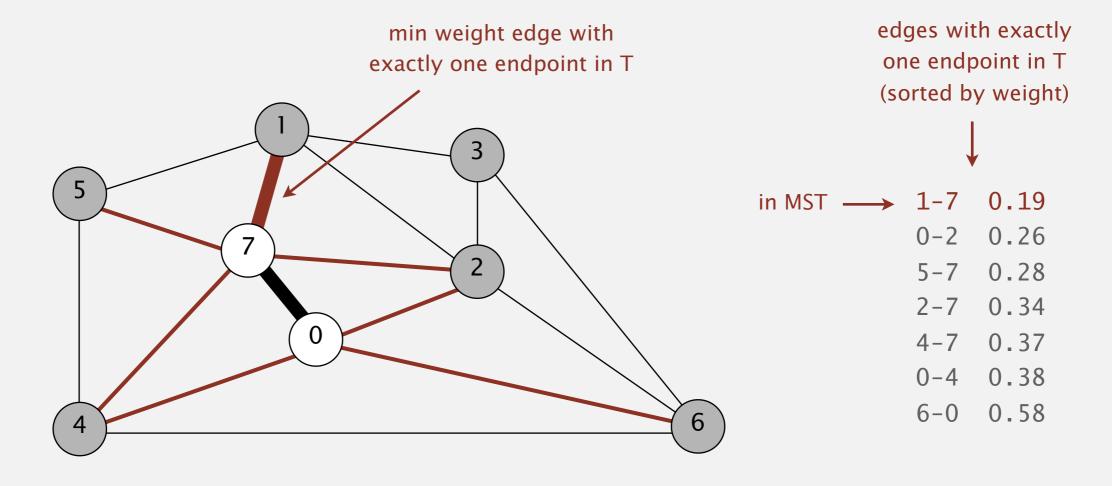


- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



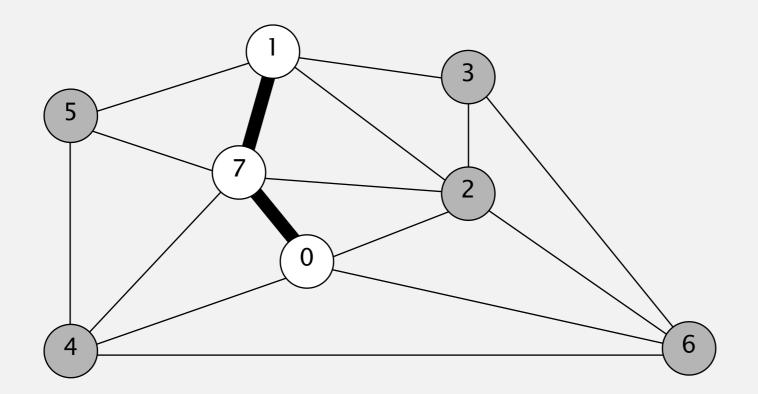
MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



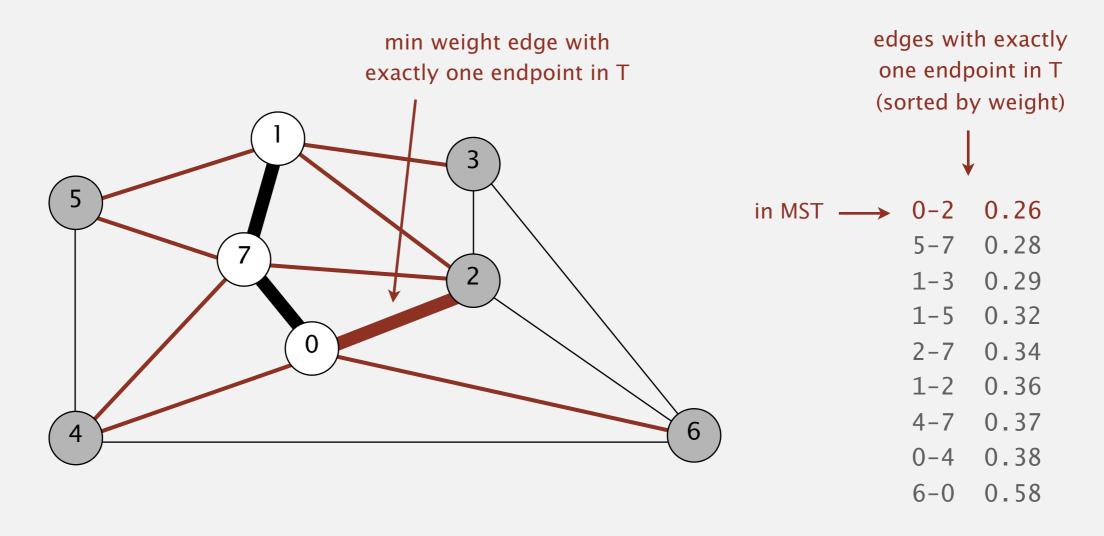
MST edges

- Start with vertex 0 and greedily grow tree *T*.
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- Repeat until V-1 edges.



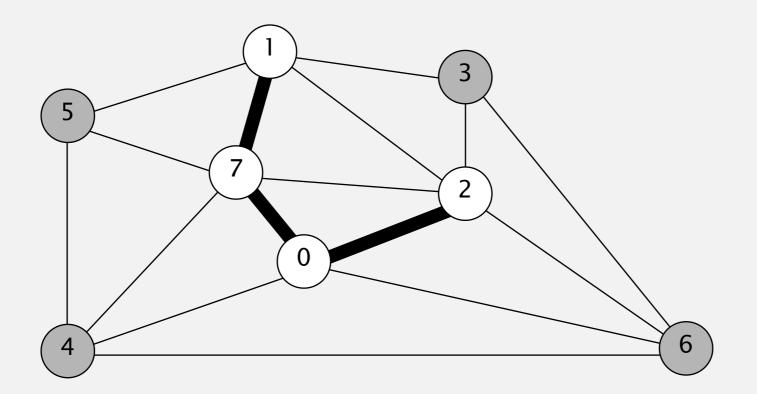
MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



MST edges

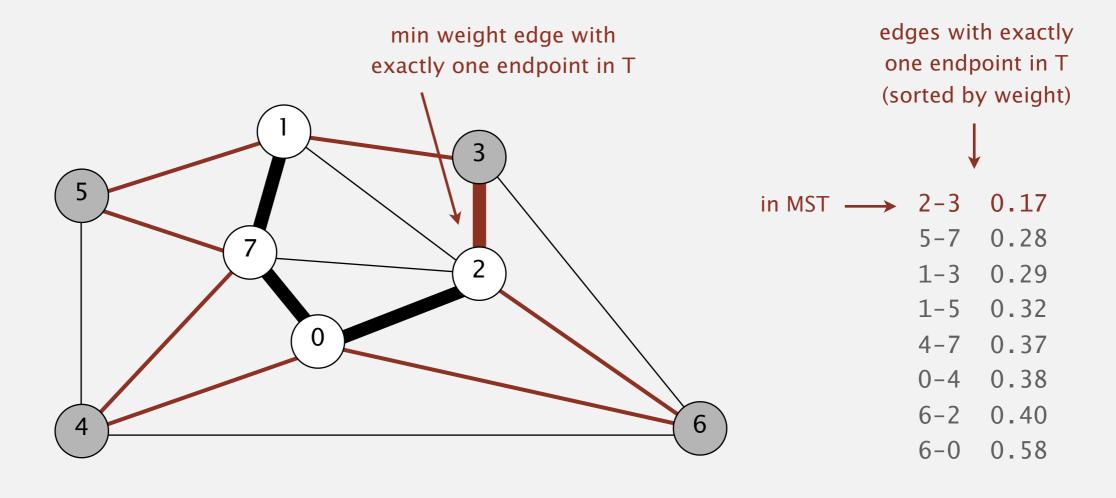
- Start with vertex 0 and greedily grow tree *T*.
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- Repeat until V-1 edges.



MST edges

0-7 1-7 0-2

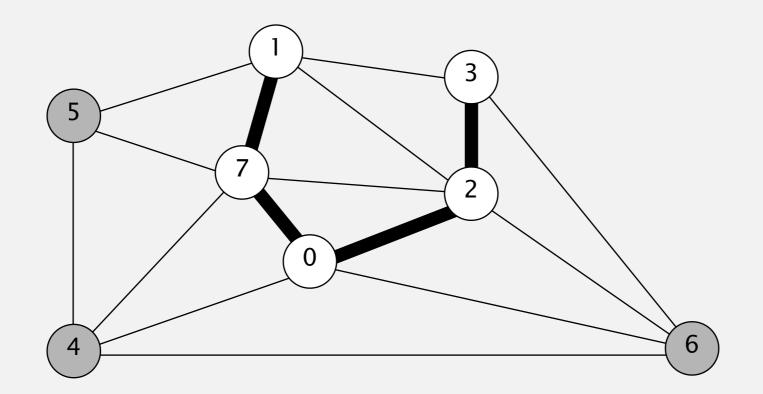
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



MST edges

0-7 1-7 0-2

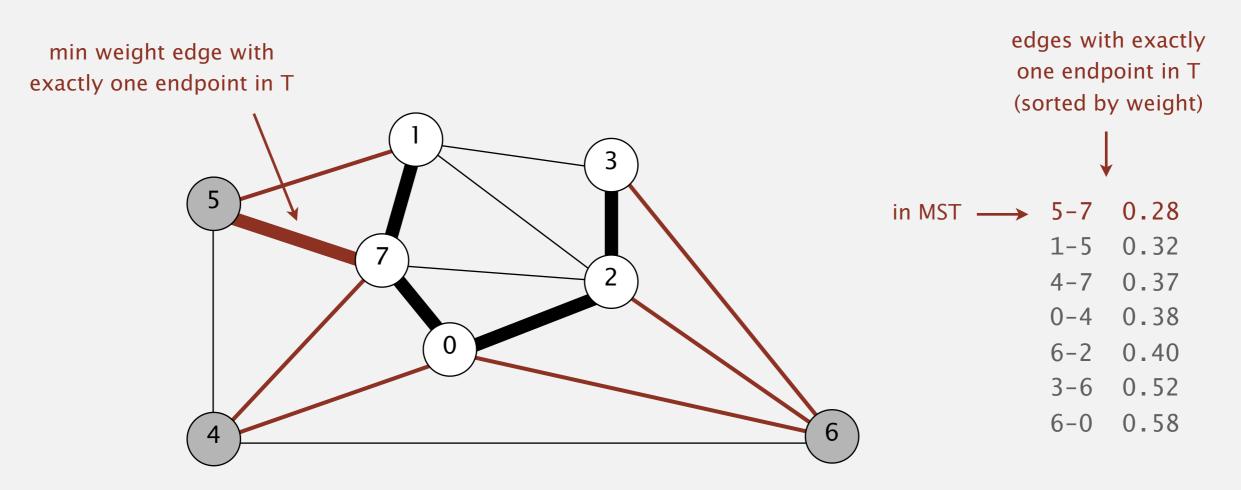
- Start with vertex 0 and greedily grow tree *T*.
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MST edges

0-7 1-7 0-2 2-3

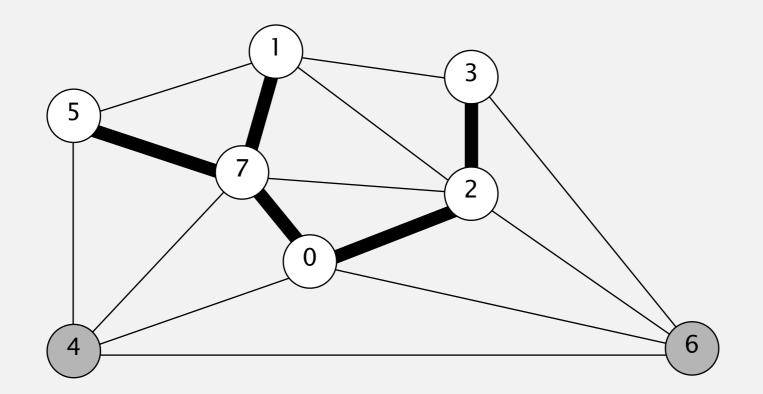
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



MST edges

0-7 1-7 0-2 2-3

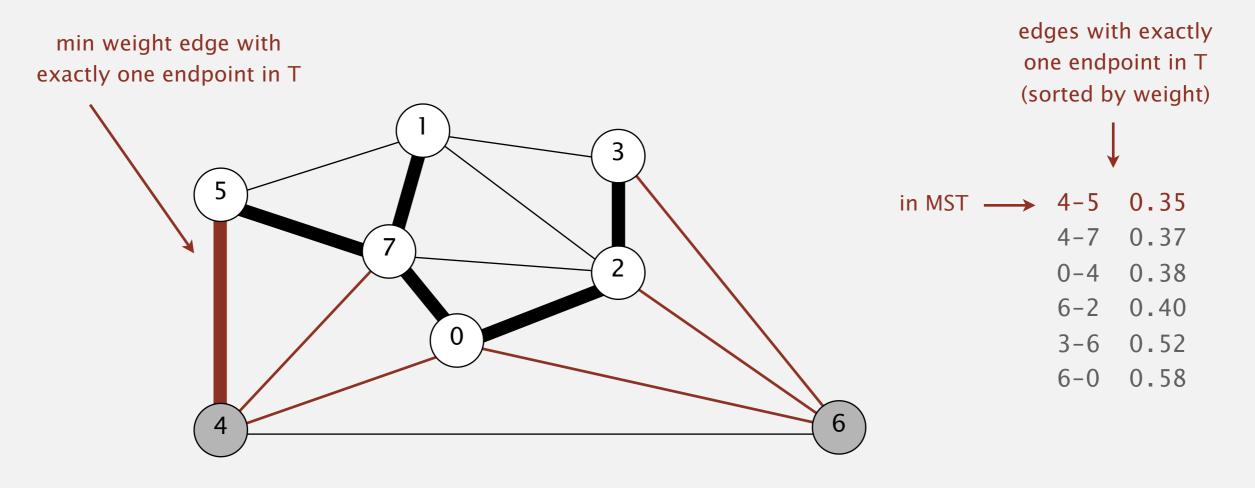
- Start with vertex 0 and greedily grow tree *T*.
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- Repeat until V-1 edges.



MST edges

0-7 1-7 0-2 2-3 5-7

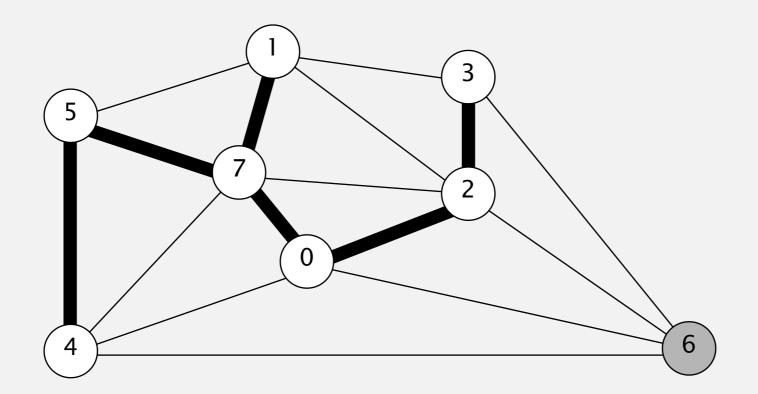
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



MST edges

0-7 1-7 0-2 2-3 5-7

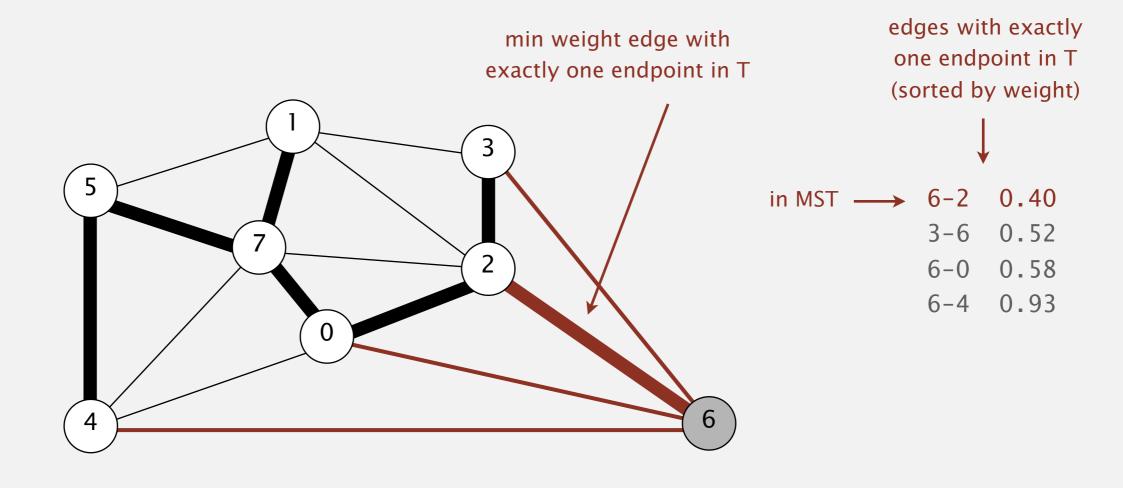
- Start with vertex 0 and greedily grow tree *T*.
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- Repeat until V-1 edges.



MST edges

0-7 1-7 0-2 2-3 5-7 4-5

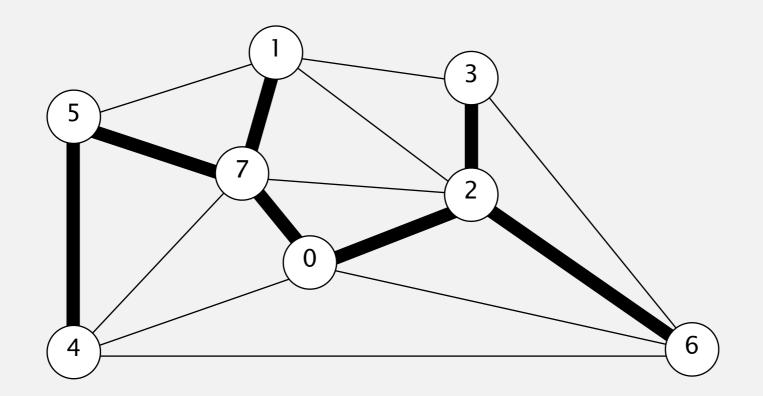
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



MST edges

0-7 1-7 0-2 2-3 5-7 4-5

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
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MST edges

0-7 1-7 0-2 2-3 5-7 4-5 6-2

PRIM'S ALGORITHM DEMO

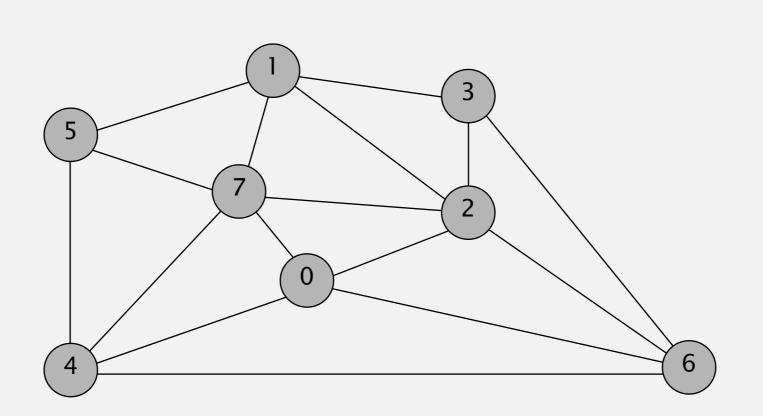
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Algorithms

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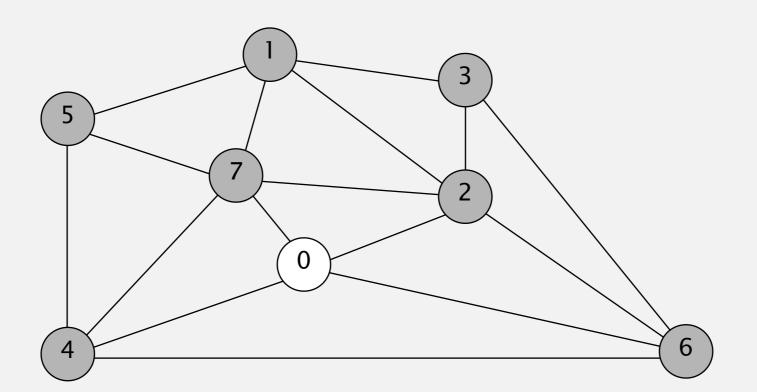
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



an edge-weighted graph

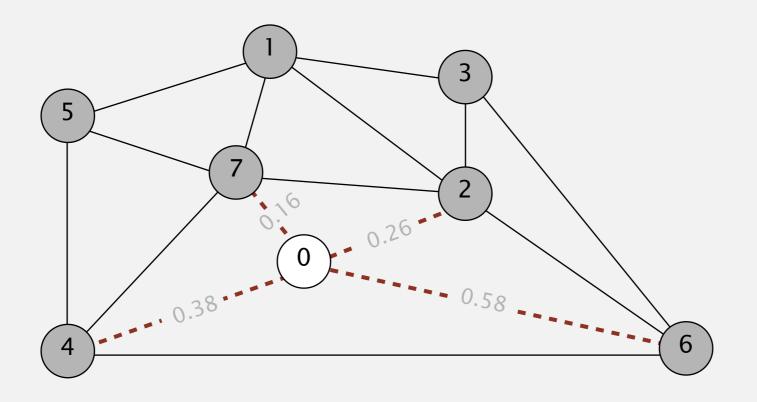
0-7 0.16 0.17 1-7 0.19 0-2 0.26 5-7 0.28 0.29 1-3 1-5 0.32 2-7 0.34 4-5 0.35 1-2 0.36 4-7 0.37 0.38 0.40 6-2 3-6 0.52 6-0 0.58 $6-4 \quad 0.93$

- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

add to PQ all edges incident to 0



edges on PQ (sorted by weight)

***** 0-7 0.16

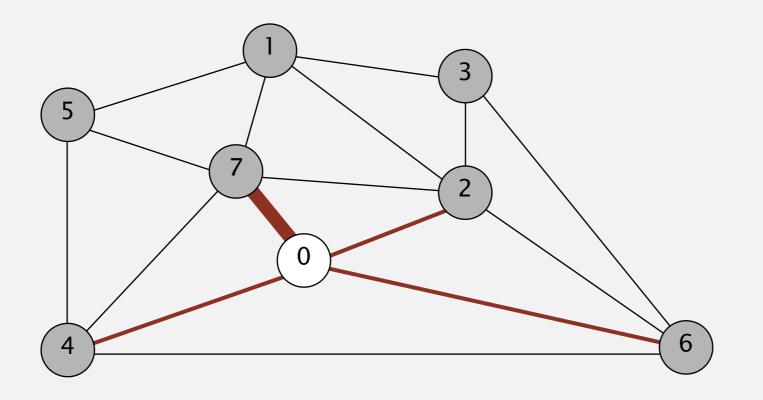
* 0-2 0.26

***** 0-4 0.38

***** 6-0 0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 0-7 and add to MST



edges on PQ (sorted by weight)

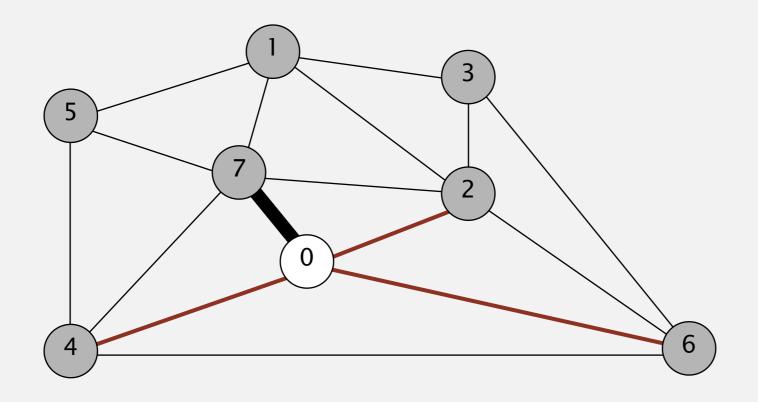
0-7 0.16

0-2 0.26

0-4 0.38

6-0 0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

0-2 0.26

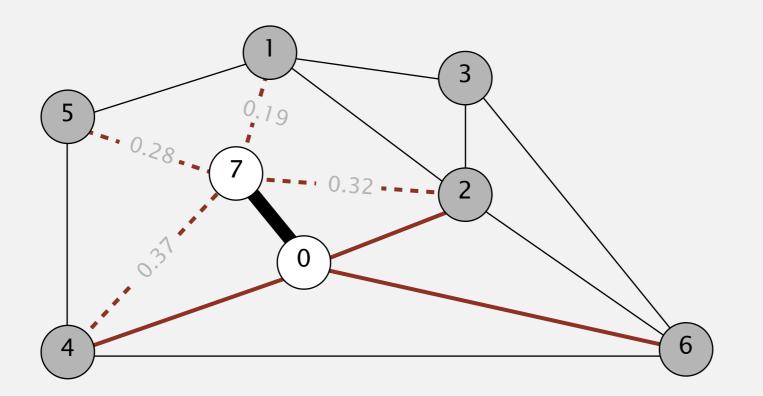
0-4 0.38

6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

add to PQ all edges incident to 7



edges on PQ (sorted by weight)

***** 1-7 0.19

0-2 0.26

***** 5-7 0.28

***** 2-7 0.34

***** 4-7 0.37

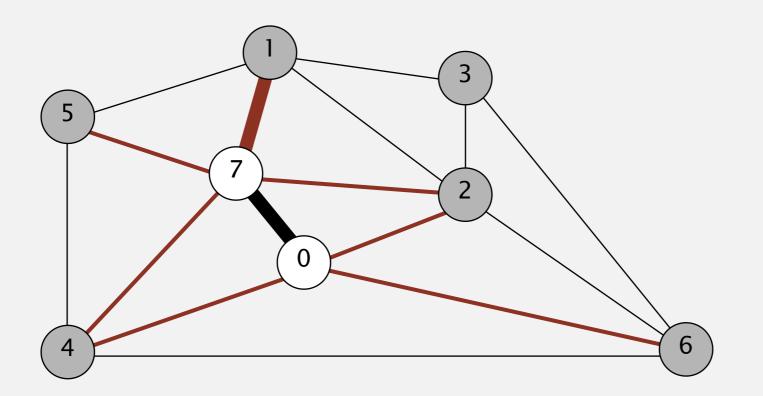
0-4 0.38

6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 1-7 and add to MST



edges on PQ (sorted by weight)

1-7 0.19 0-2 0.26

5-7 0.28

2-7 0.34

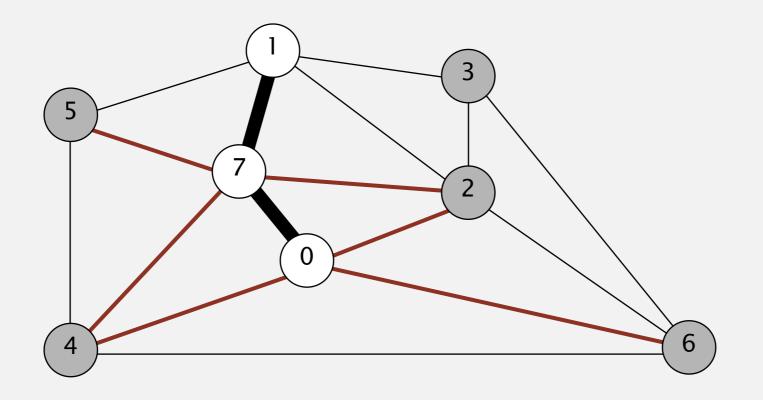
4-7 0.37

0-4 0.38

6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

0-2 0.26

5-7 0.28

2-7 0.34

4-7 0.37

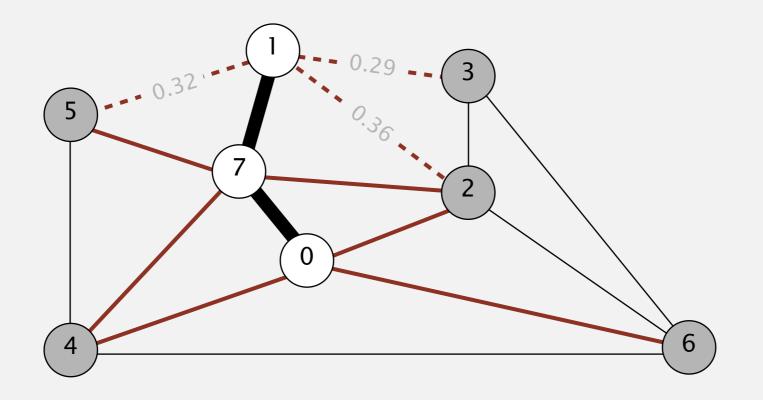
0-4 0.38

6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

add to PQ all edges incident to 1



edges on PQ (sorted by weight)

0-2 0.26

5-7 0.28

***** 1-3 0.29

***** 1-5 0.32

2-7 0.34

***** 1-2 0.36

4-7 0.37

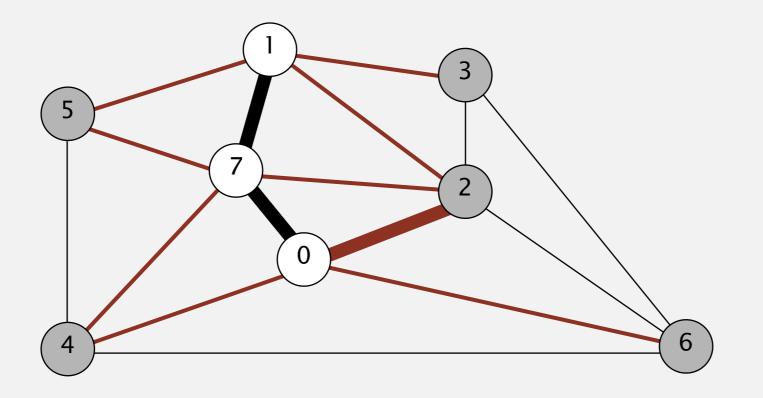
0-4 0.38

6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete edge 0-2 and add to MST



edges on PQ (sorted by weight)

0-2 0.26

5-7 0.28

1-3 0.29

1-5 0.32

2-7 0.34

1-2 0.36

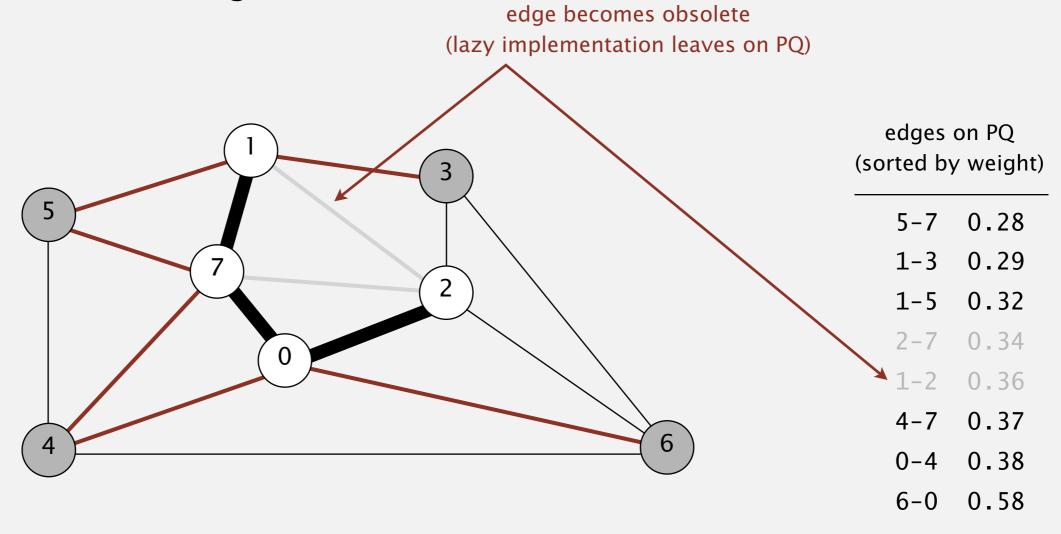
4-7 0.37

0-4 0.38

6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



MST edges

0-7 1-7 0-2

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.

no need to add edge 1-2 or 2-7

• Repeat until V-1 edges.

add to PQ all edges incident to 2

because it's already obsolete

7

0.17

7

0.17

6

MST edges

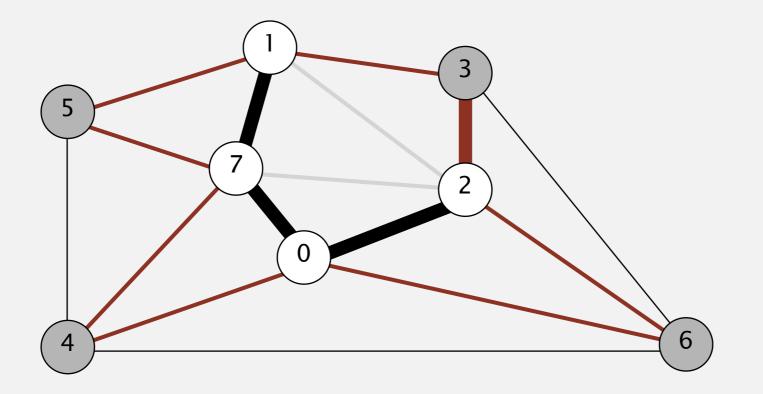
0-7 1-7 0-2

edges on PQ (sorted by weight)

| * | 2-3 | 0.17 |
|---|-----|------|
| | 5-7 | 0.28 |
| | 1-3 | 0.29 |
| | 1-5 | 0.32 |
| | 2-7 | 0.34 |
| | 1-2 | 0.36 |
| | 4-7 | 0.37 |
| | 0-4 | 0.38 |
| * | 6-2 | 0.40 |
| | 6-0 | 0.58 |

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 2-3 and add to MST



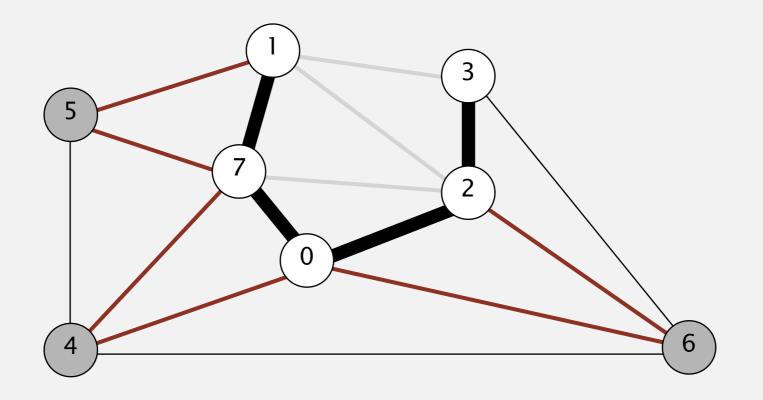
MST edges

0-7 1-7 0-2

edges on PQ (sorted by weight)

| * 2-3 | 0.17 |
|--------------|------|
| 5-7 | 0.28 |
| 1-3 | 0.29 |
| 1-5 | 0.32 |
| 2-7 | 0.34 |
| 1-2 | 0.36 |
| 4-7 | 0.37 |
| 0-4 | 0.38 |
| * 6-2 | 0.40 |
| 6-0 | 0.58 |

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

5-7 0.28

 $1-3 \quad 0.29$

1-5 0.32

2-7 0.34

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

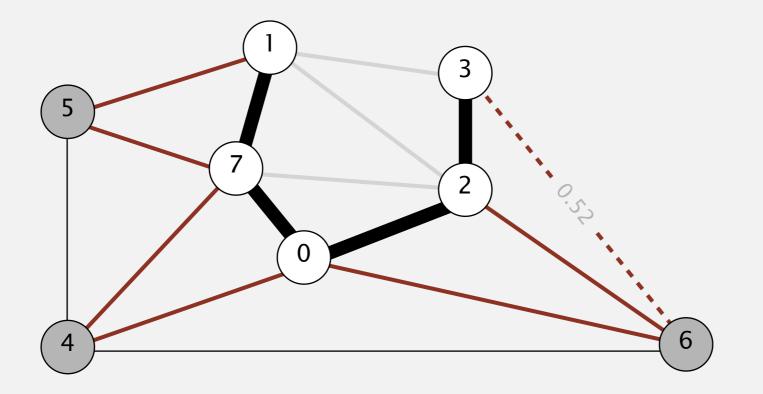
6-0 0.58

MST edges

0-7 1-7 0-2 2-3

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

add to PQ all edges incident to 3



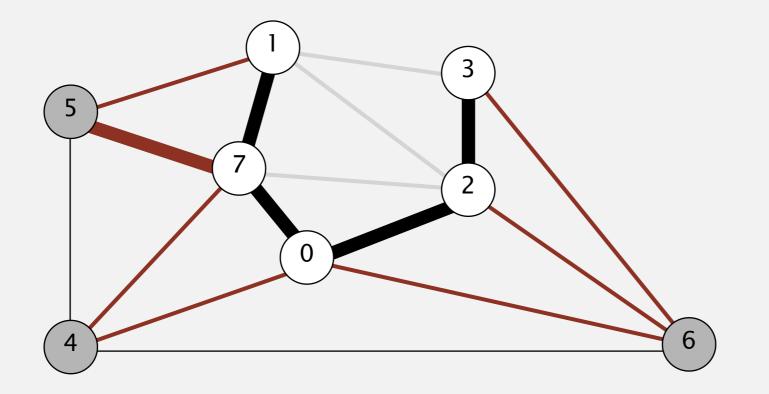
MST edges

0-7 1-7 0-2 2-3

edges on PQ (sorted by weight)

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 5-7 and add to MST



MST edges

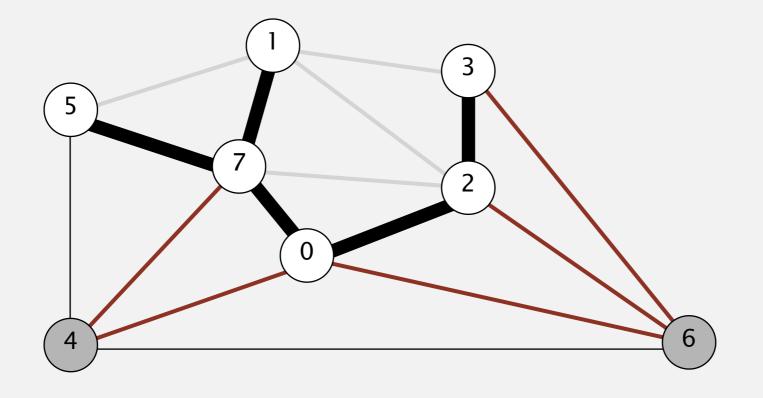
0-7 1-7 0-2 2-3

edges on PQ (sorted by weight)

| 5-7 | 0.28 |
|------------|------|
| <i>J</i> , | 0.20 |

$$6-0$$
 0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

1-3 0.29

1-5 0.32

2-7 0.34

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

3-6 0.52

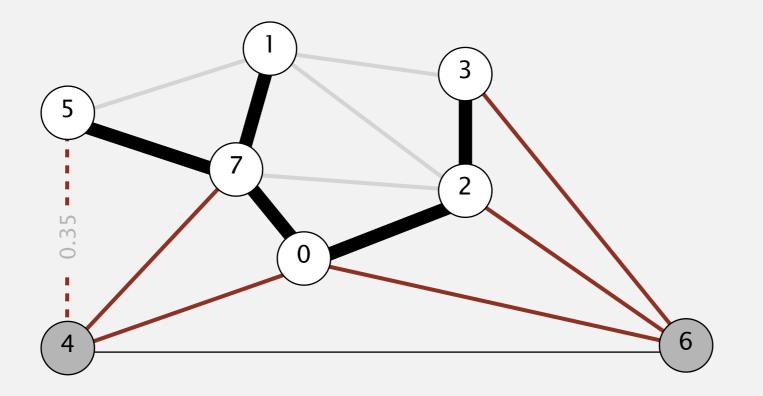
6-0 0.58

MST edges

0-7 1-7 0-2 2-3 5-7

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

add to PQ all edges incident to 5



MST edges

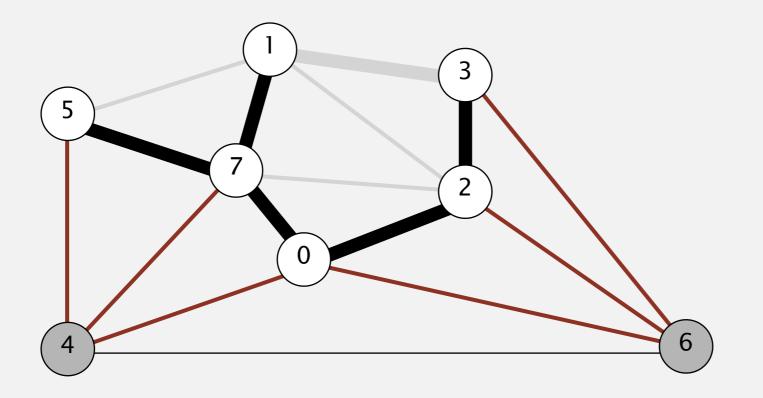
0-7 1-7 0-2 2-3 5-7

edges on PQ (sorted by weight)

| 1-3 | 0.29 |
|--------------|------|
| 1-5 | 0.32 |
| 2-7 | 0.34 |
| * 4-5 | 0.35 |
| 1-2 | 0.36 |
| 4-7 | 0.37 |
| 0-4 | 0.38 |
| 6-2 | 0.40 |
| 3-6 | 0.52 |
| 6-0 | 0.58 |

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 1-3 and discard obsolete edge



MST edges

0-7 1-7 0-2 2-3 5-7

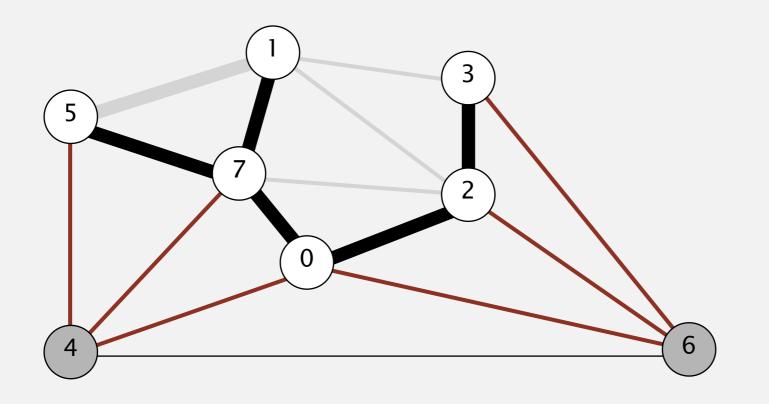
edges on PQ (sorted by weight)

| 1_ | 3 | \cap | 7 | (|
|----|---|--------|---|---|
| |) | U | | |

$$6-0$$
 0.58

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 1-5 and discard obsolete edge



edges on PQ (sorted by weight)

1-5 0.32

2-7 0.34

4-5 0.35

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

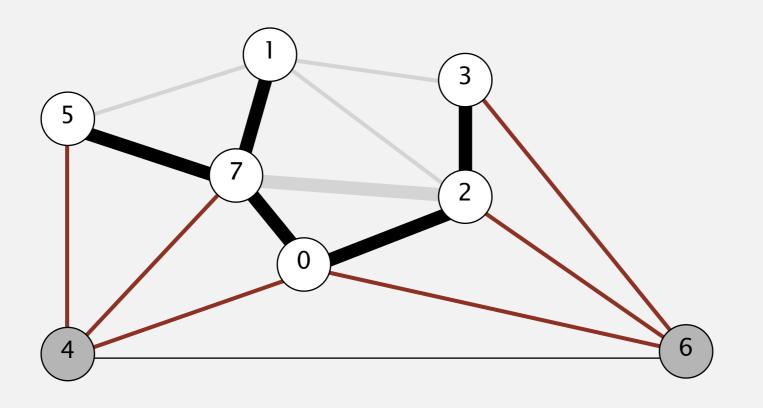
3-6 0.52

6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 2-7 and discard obsolete edge



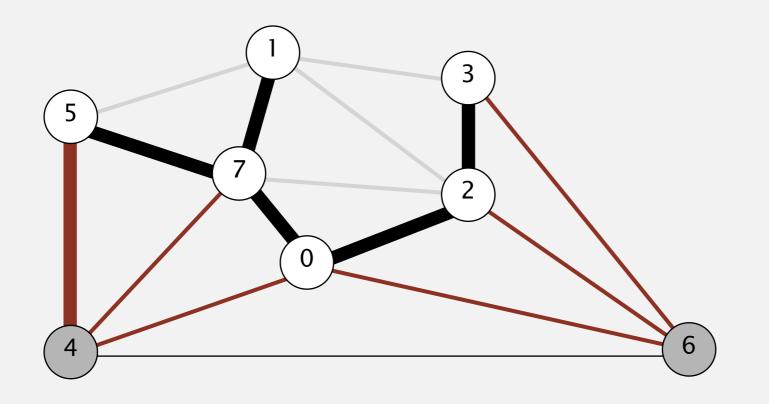
edges on PQ (sorted by weight)

2-7 0.34 4-5 0.35 1-2 0.36 4-7 0.37 0-4 0.38 6-2 0.40 3-6 0.52 6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 4-5 and add to MST



edges on PQ (sorted by weight)

4-5 0.35

1-2 0.36

4-7 0.37

0-4 0.38

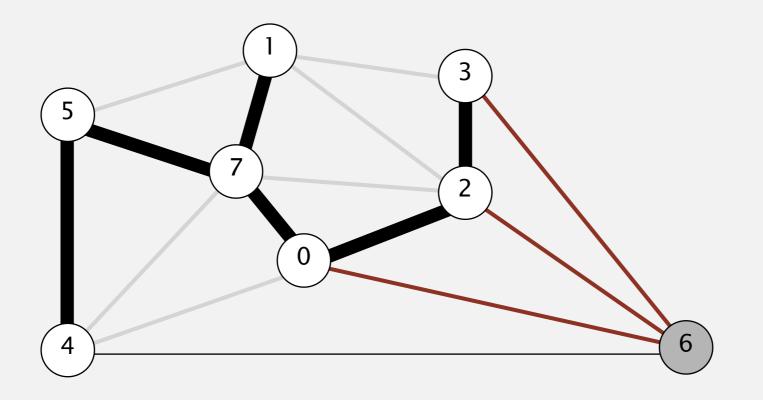
6-2 0.40

3-6 0.52

6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



edges on PQ (sorted by weight)

1-2 0.36

4-7 0.37

0-4 0.38

6-2 0.40

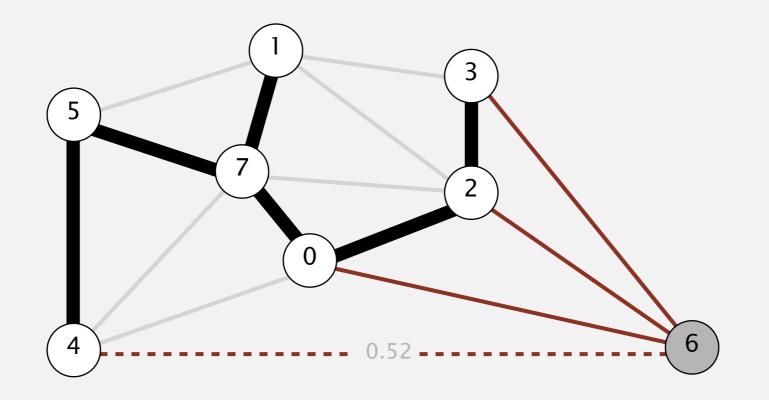
3-6 0.52

6-0 0.58

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

add to PQ all edges incident to 4



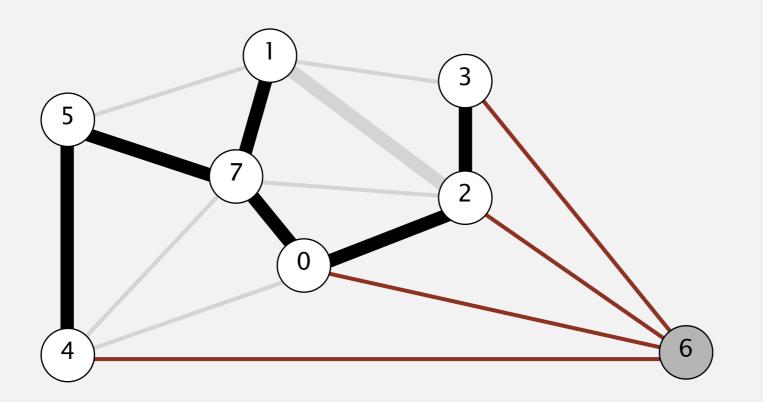
edges on PQ (sorted by weight)

| 1-2 | 0.36 |
|-----|------|
| 4-7 | 0.37 |
| 0-4 | 0.38 |
| 6-2 | 0.40 |
| 3-6 | 0.52 |
| 6-0 | 0.58 |
| | |

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 1-2 and discard obsolete edge



edges on PQ (sorted by weight)

1-2 0.36 4-7 0.37

0-4 0.38

6-2 0.40

3-6 0.52

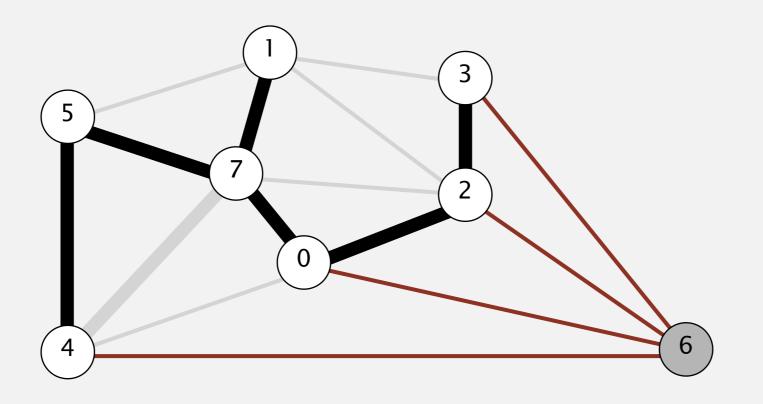
6-0 0.58

6-4 0.93

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 4-7 and discard obsolete edge



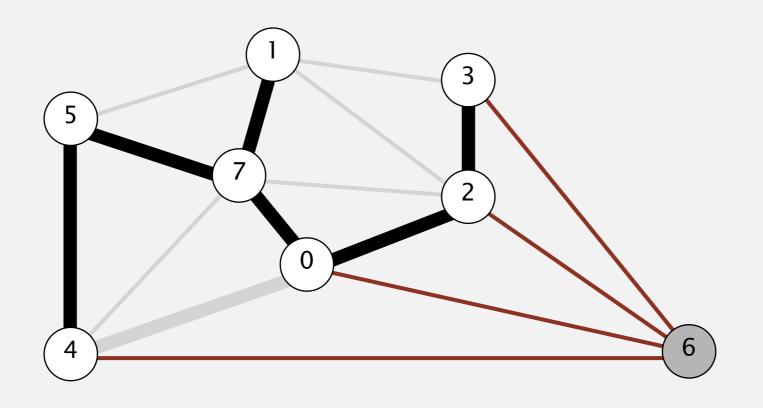
edges on PQ (sorted by weight)

4-7 0.37 0-4 0.38 6-2 0.40 3-6 0.52 6-0 0.58 6-4 0.93

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 0-4 and discard obsolete edge



edges on PQ (sorted by weight)

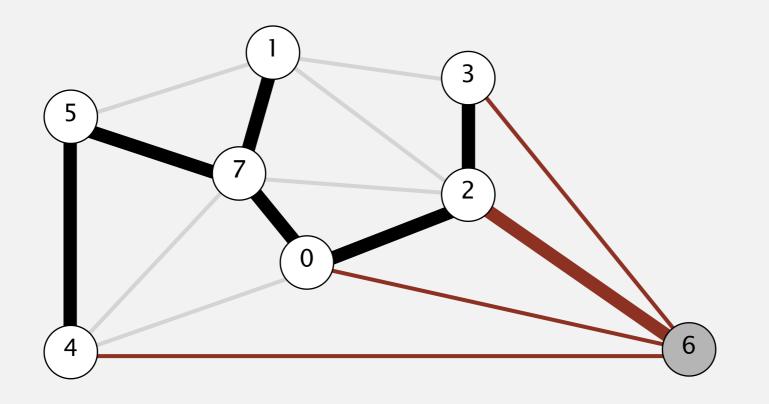
0-4 0.38 6-2 0.40 3-6 0.52 6-0 0.58

6-4 0.93

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 6-2 and add to MST



edges on PQ (sorted by weight)

6-2 0.40

3-6 0.52

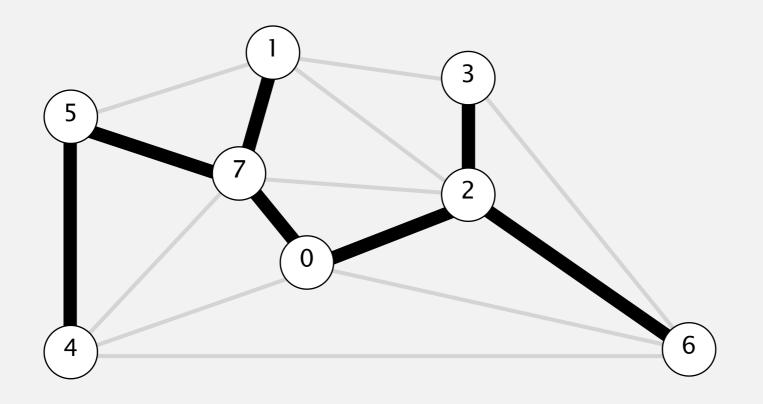
6-0 0.58

6-4 0.93

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

delete 6-2 and add to MST



edges on PQ (sorted by weight)

3-6 0.52

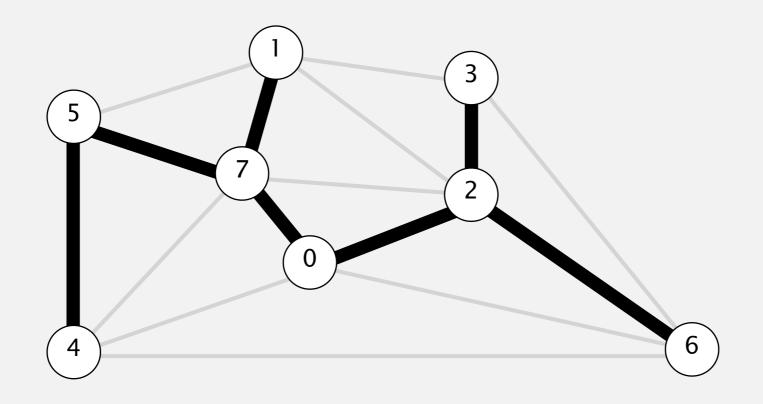
6-0 0.58

 $6-4 \quad 0.93$

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.

stop since V-1 edges



edges on PQ (sorted by weight)

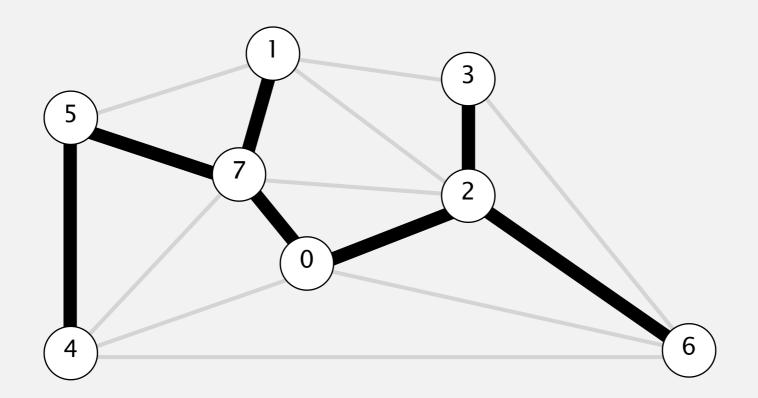
3-6 0.52

6-0 0.58

 $6-4 \quad 0.93$

MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



MST edges

PRIM'S ALGORITHM DEMO

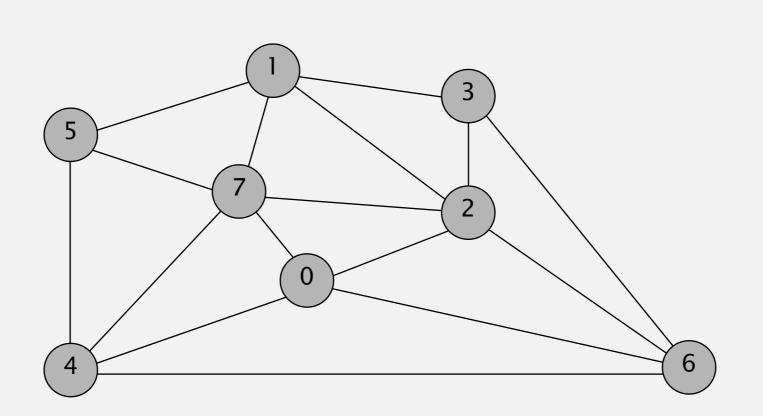
- Prim's algorithm
- lazy implementation
- eager implementation

Algorithms

ROBERT SEDGEWICK | KEVIN WAYNE

http://algs4.cs.princeton.edu

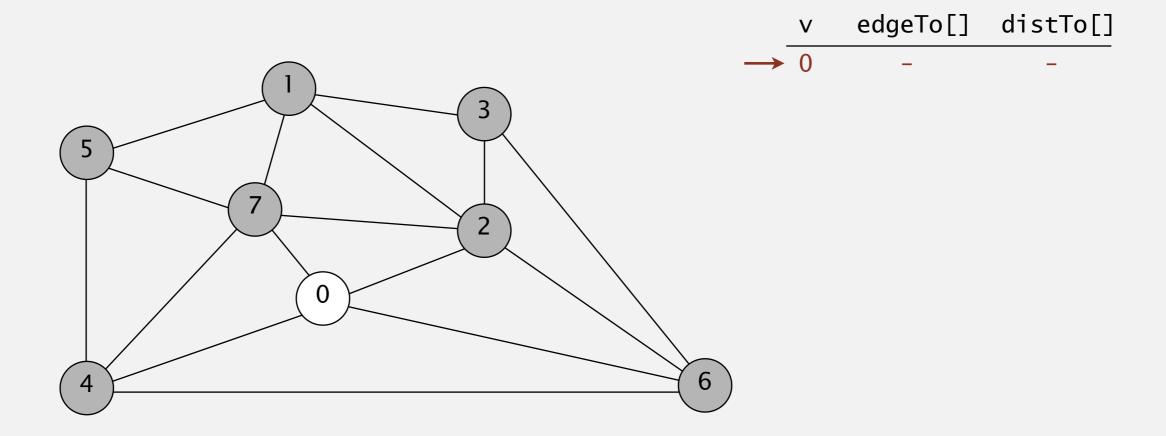
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



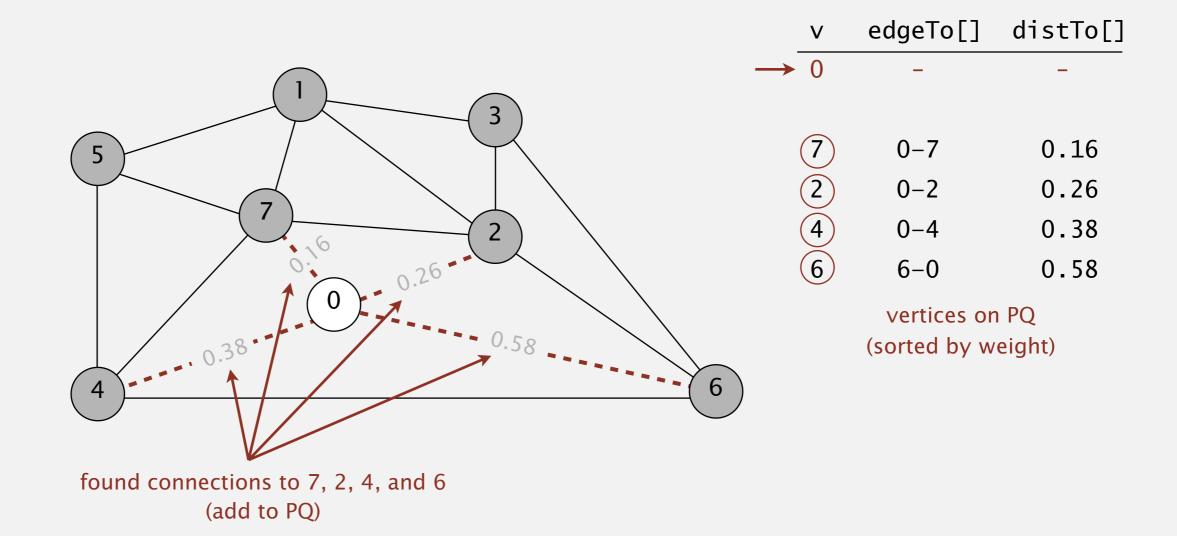
an edge-weighted graph

0-7 0.16 0.17 1-7 0.19 0-2 0.26 5-7 0.28 0.29 1-3 1-5 0.32 2-7 0.34 4-5 0.35 1-2 0.36 4-7 0.37 0.38 6-2 0.40 3-6 0.52 6-0 0.58 $6-4 \quad 0.93$

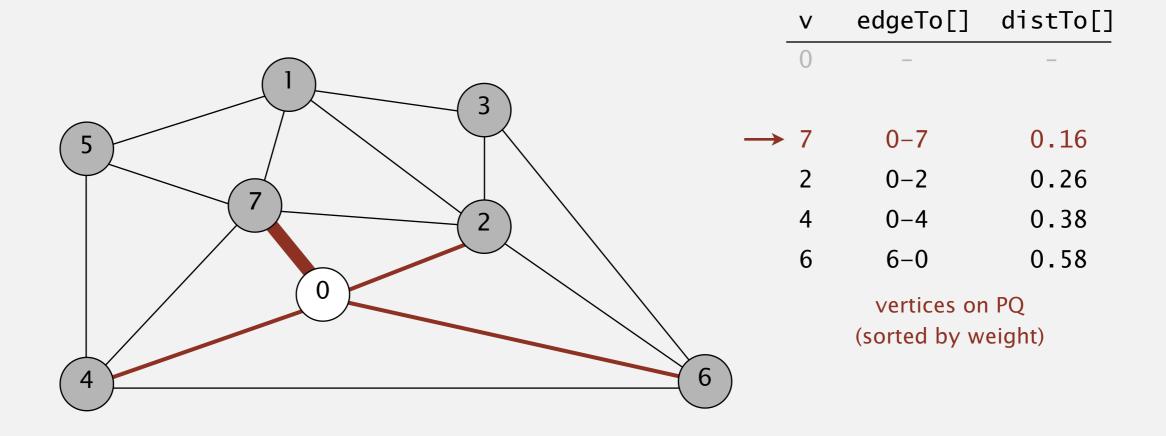
- Start with vertex 0 and greedily grow tree *T*.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



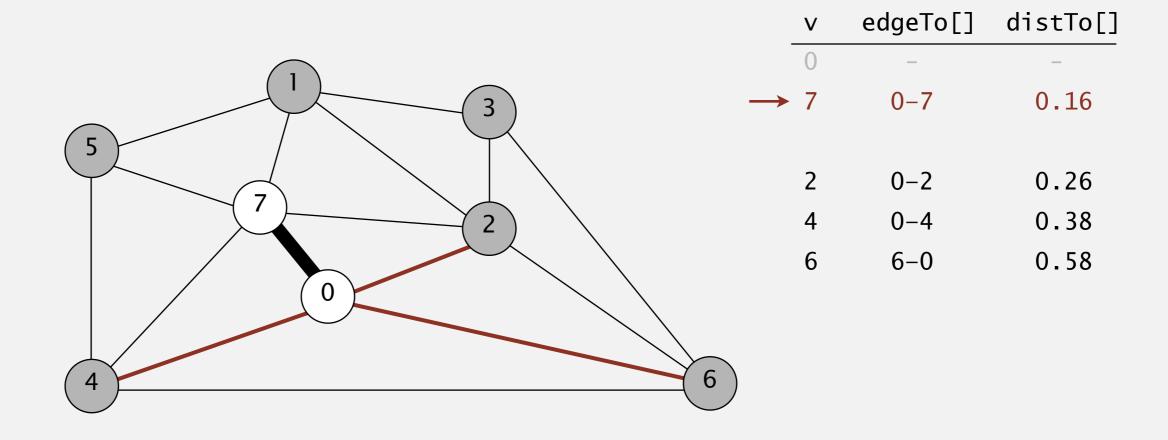
- Start with vertex 0 and greedily grow tree T.
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- Repeat until V-1 edges.



- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



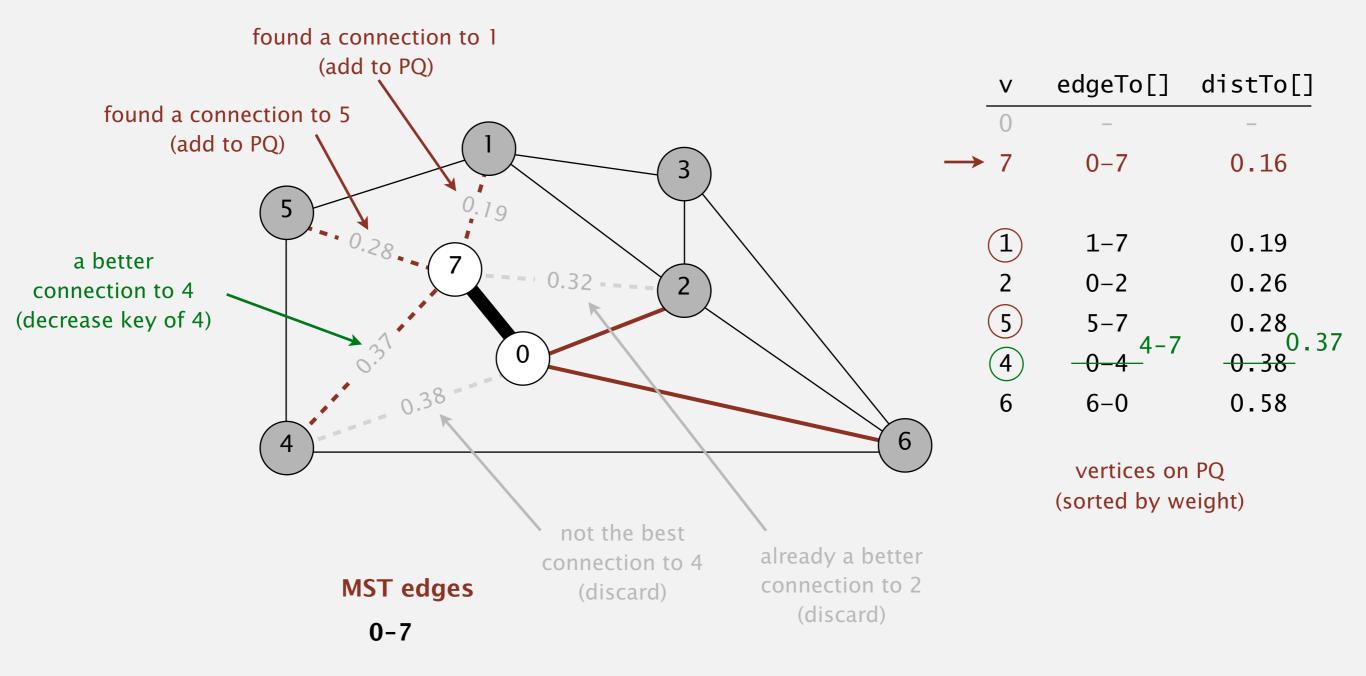
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



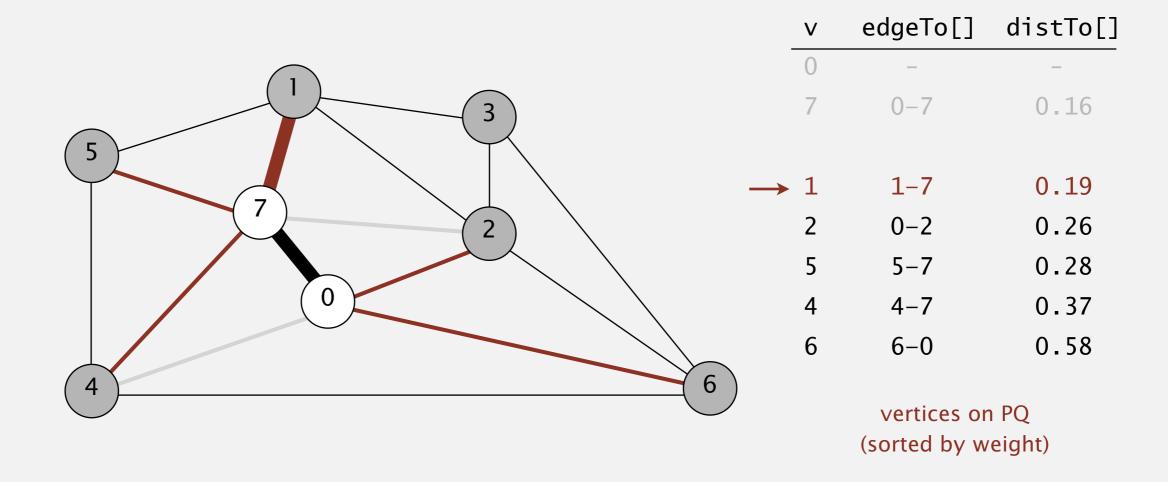
MST edges

0-7

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



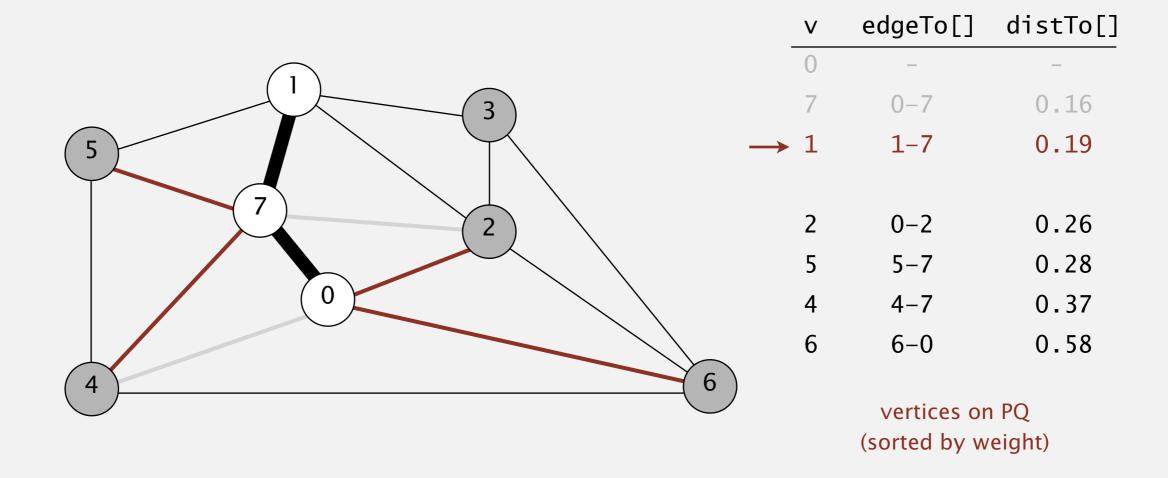
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



MST edges

0-7 1-7

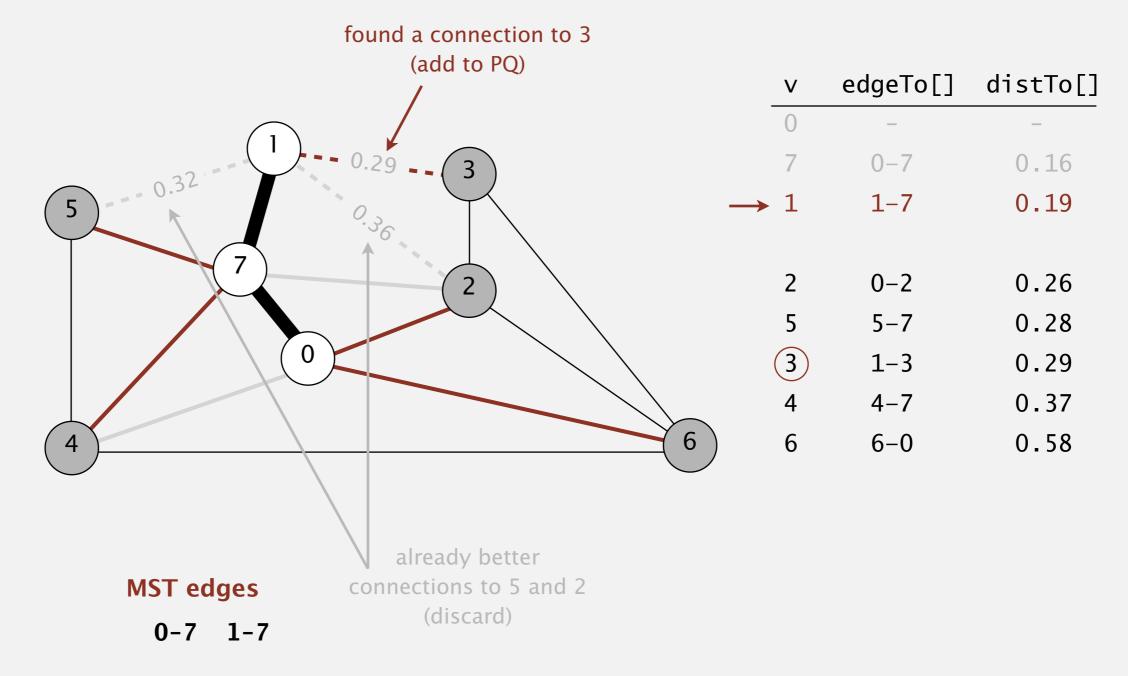
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



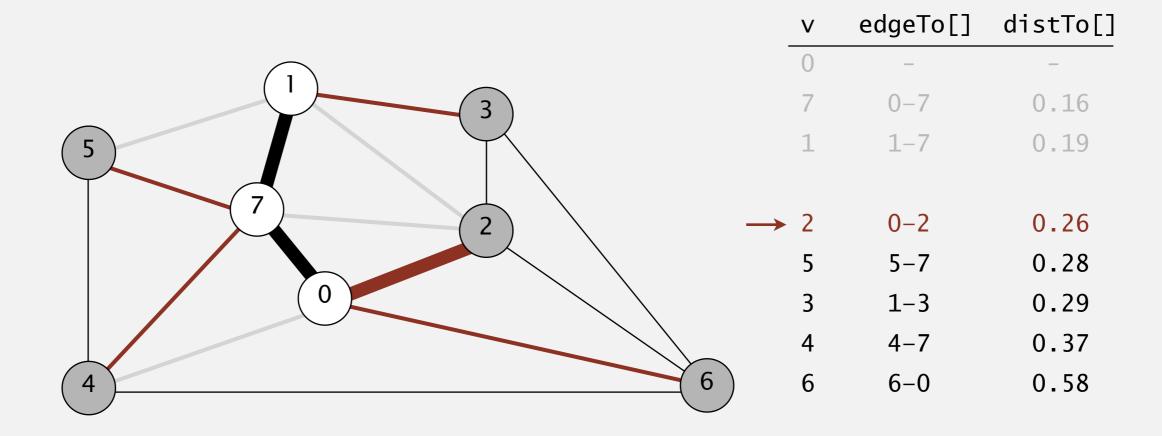
MST edges

0-7 1-7

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



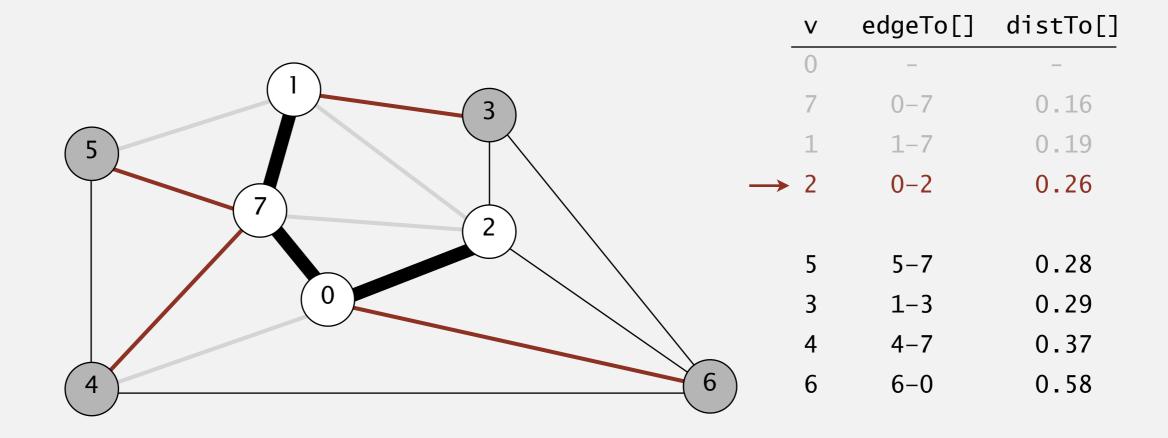
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



MST edges

0-7 1-7

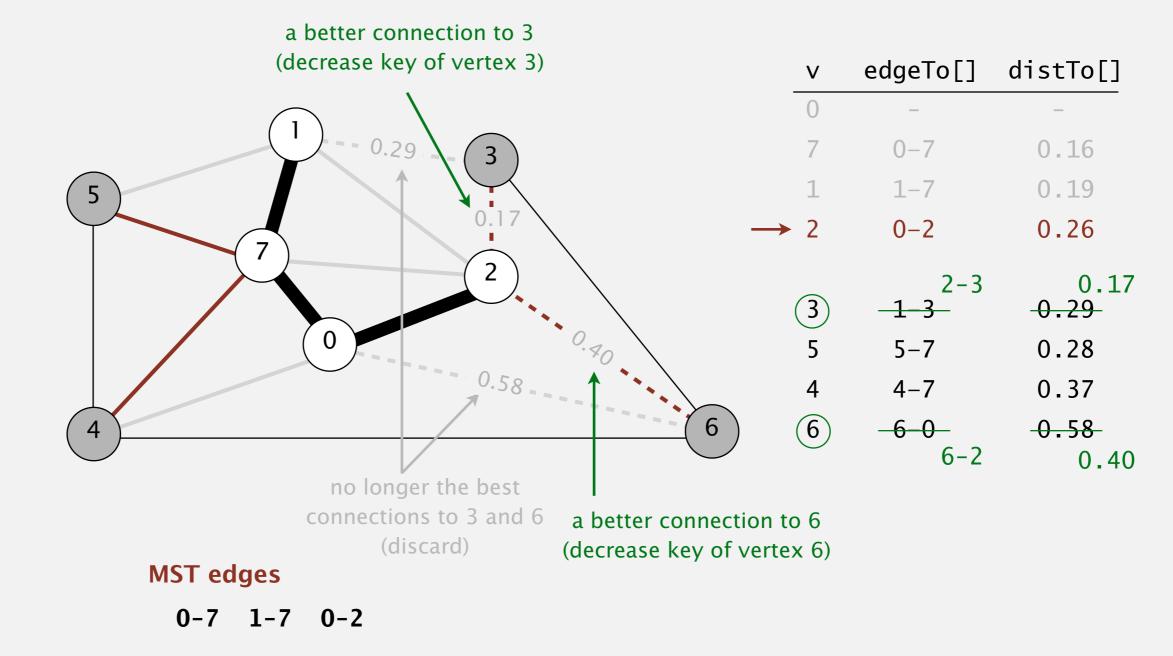
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



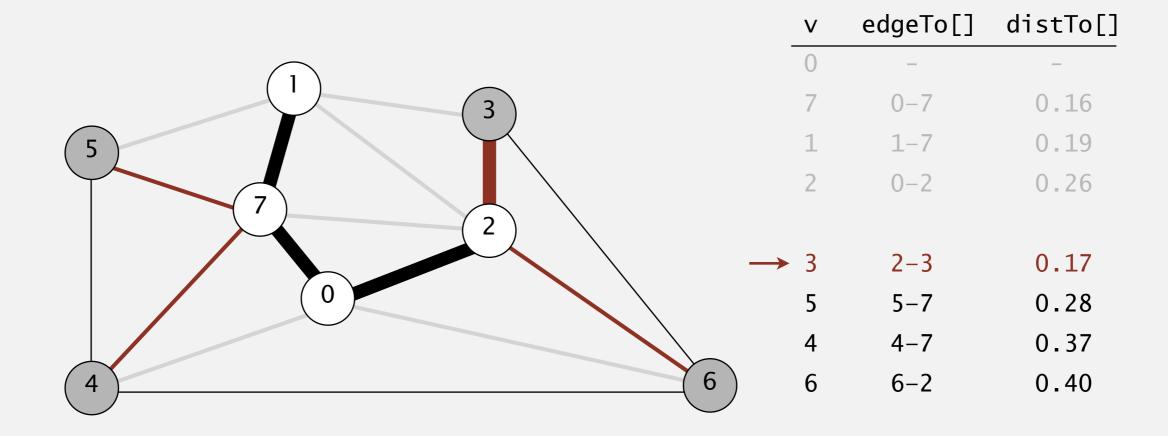
MST edges

0-7 1-7 0-2

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



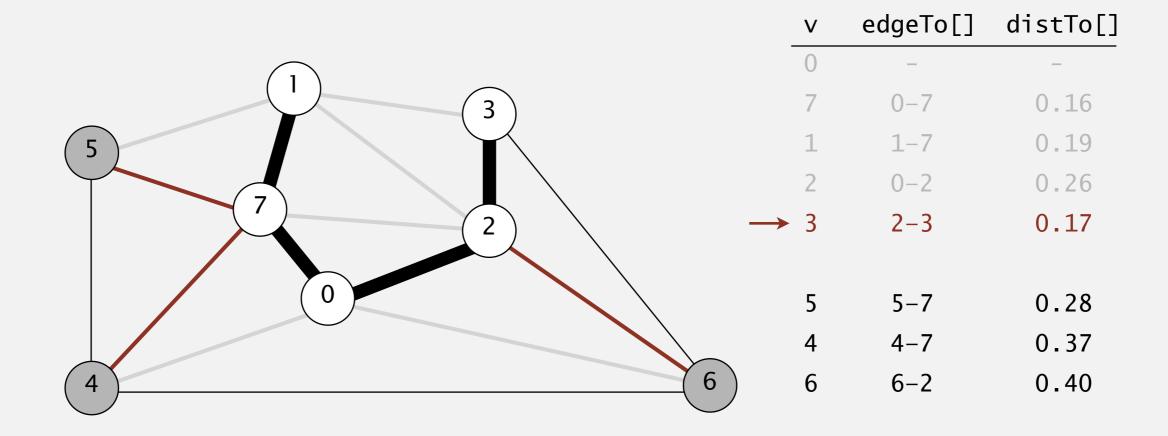
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



MST edges

0-7 1-7 0-2 2-3

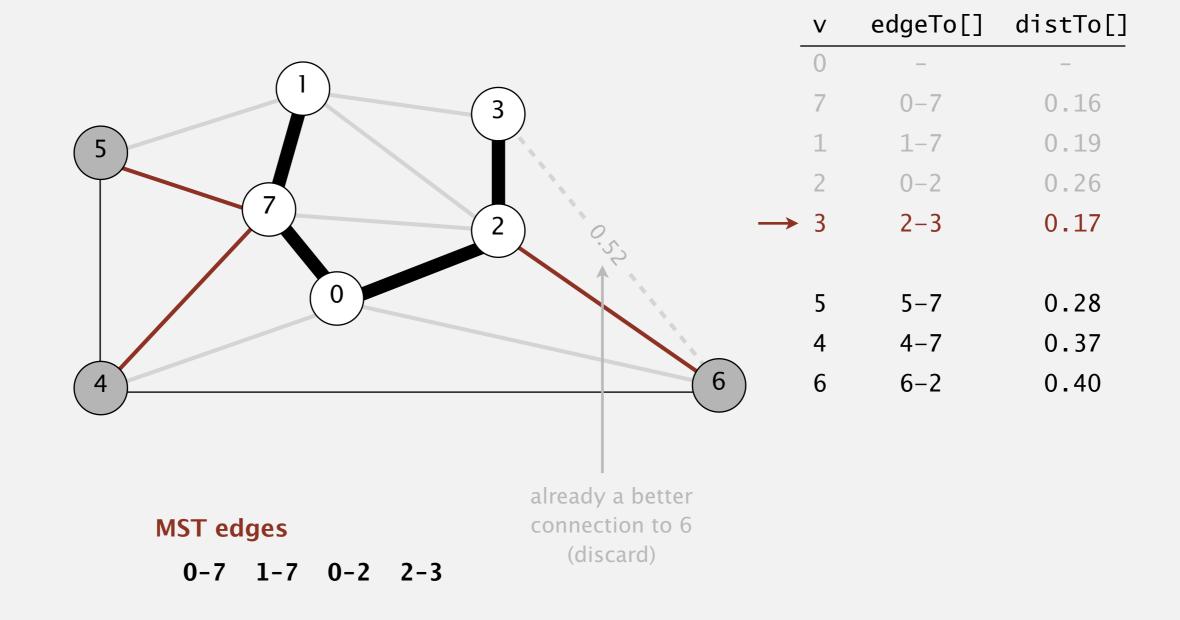
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



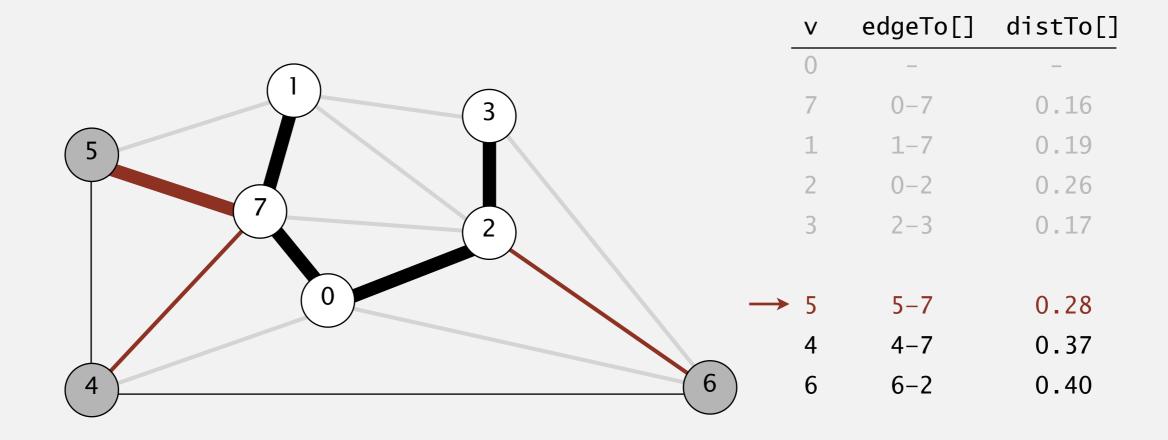
MST edges

0-7 1-7 0-2 2-3

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



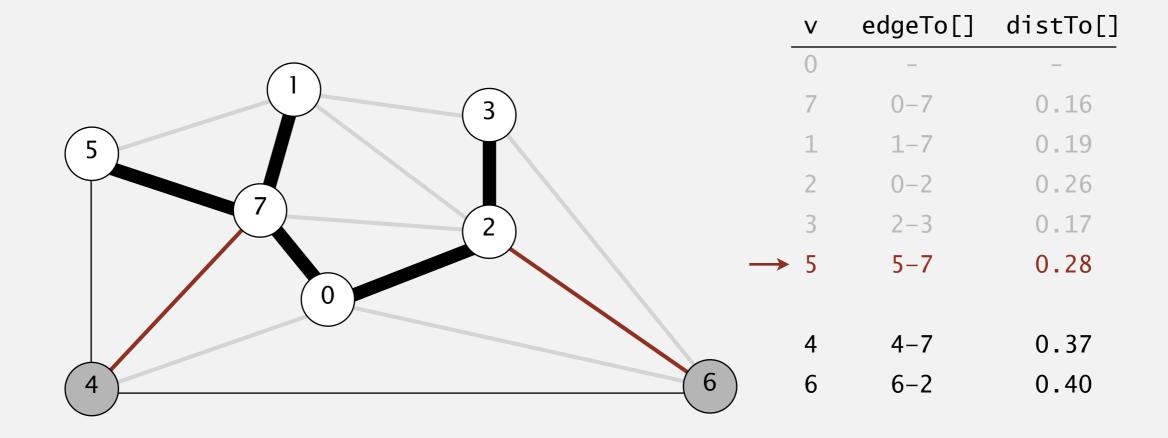
- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



MST edges

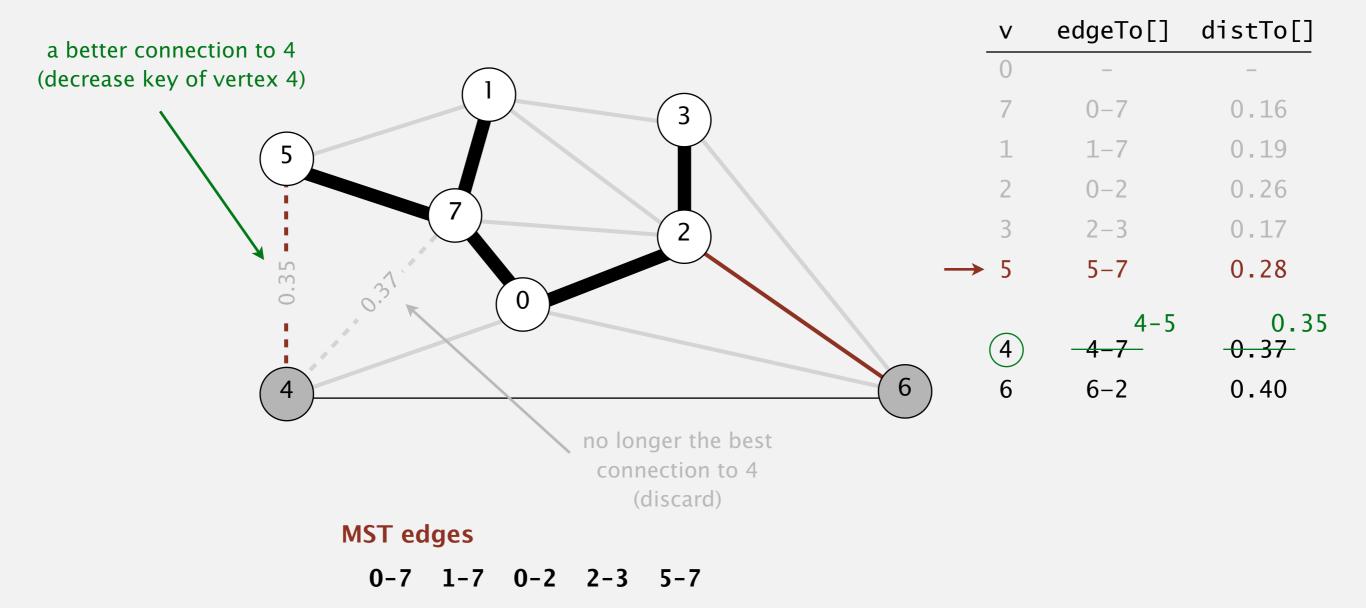
0-7 1-7 0-2 2-3

- Start with vertex 0 and greedily grow tree T.
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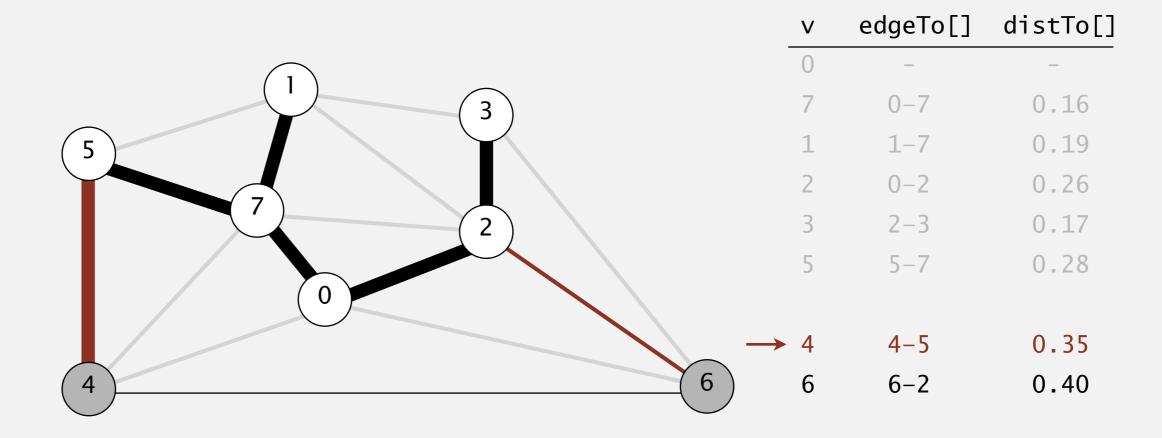


MST edges

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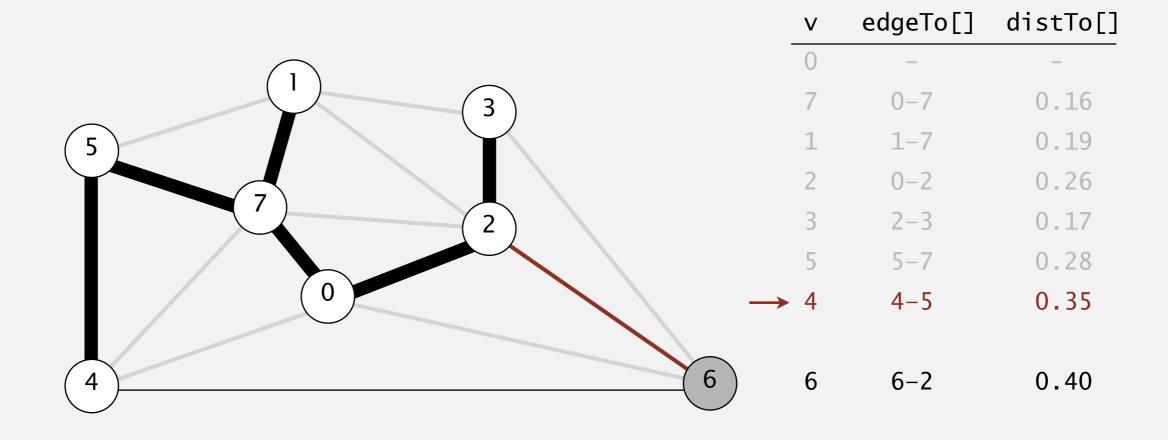


- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



MST edges

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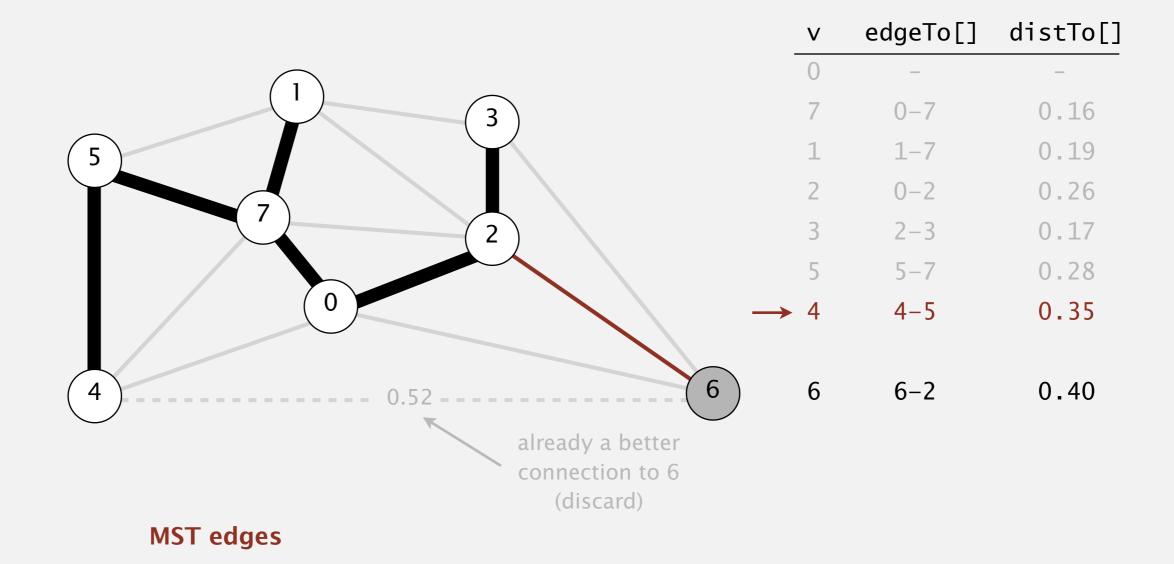


MST edges

- Start with vertex 0 and greedily grow tree T.
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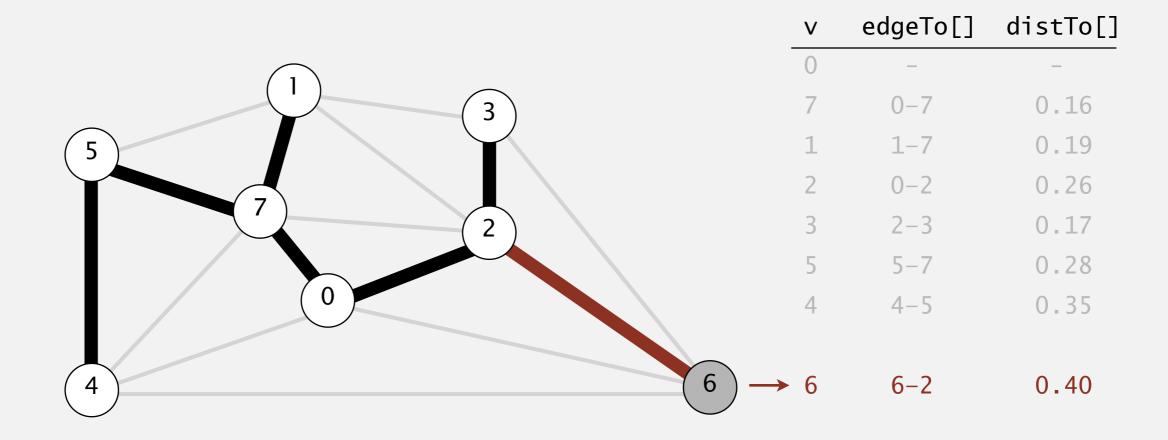
0-7 1-7 0-2 2-3 5-7 4-5

• Repeat until V-1 edges.



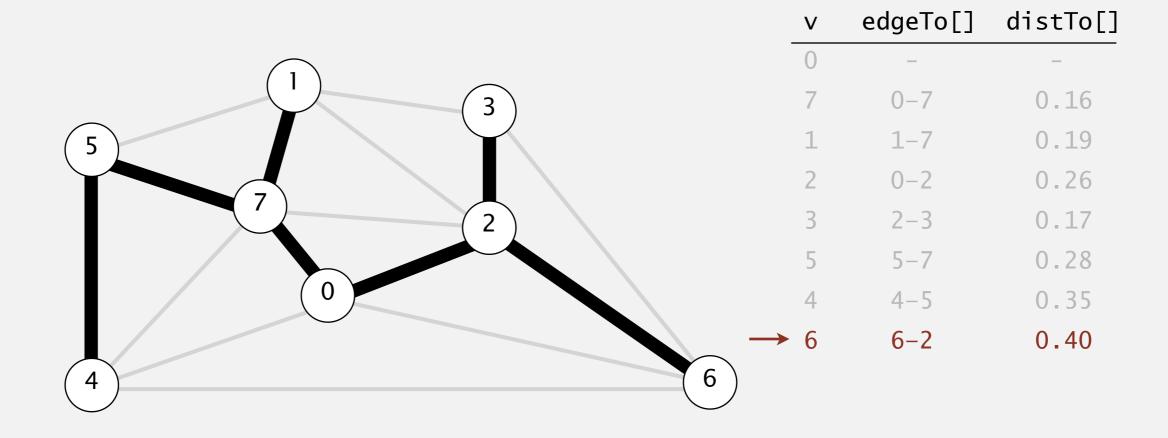
72

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



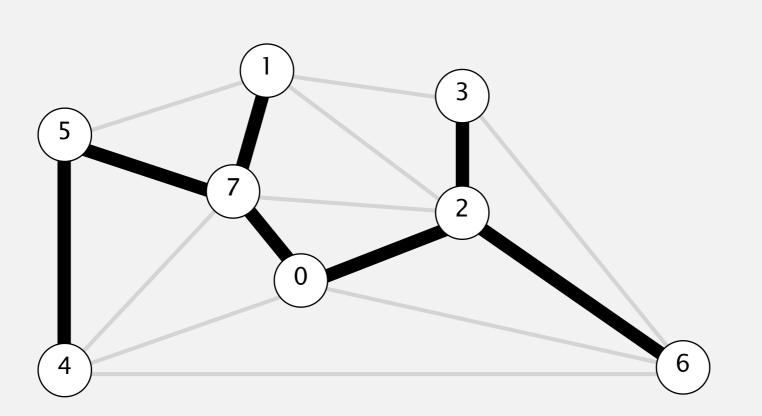
MST edges

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MST edges

- Start with vertex 0 and greedily grow tree T.
- Add to T the min weight edge with exactly one endpoint in T.
- Repeat until V-1 edges.



| V | edgeTo[] | distTo[] |
|---|----------|----------|
| 0 | - | _ |
| 7 | 0-7 | 0.16 |
| 1 | 1-7 | 0.19 |
| 2 | 0-2 | 0.26 |
| 3 | 2–3 | 0.17 |
| 5 | 5-7 | 0.28 |
| 4 | 4-5 | 0.35 |
| 6 | 6–2 | 0.40 |
| | | |

MST edges