CSC 347 - Concepts of Programming Languages

Collections Processing: Fold

Instructor: James Riely

Learning Objectives

❓ How to combine collection elements into an aggregate result?

- Express result aggregation using `fold`
- Identify the difference between `foldLeft` and `foldRight`

Exercise: Sum the Elements of a List

Express by iterating through the list

Java

```
1 int sum (List<Int> xs) {
2   int result = 0;
3   for (x : xs) result += x;
4   return result;
5 }
```

Scala

```
1 def sum (xs:List[Int]) : Int = {
2   var result = 0
3   for x <- xs do result += x
4   result
5 } ensuring { case result => ??? }
```

- Want a concise function to aggregate all elements of `xs` into a single result

Exercise: Sum the Elements of a List

Express with a recursive function

```
1 def sum (xs:List[Int])          : Int =  xs match
2   case Nil      => 0
3   case x::rest => x + sum (rest)
```

```
1 val xs = List(11,21,31)
2 sum (xs)
```

```
sum(11::21::31::Nil)
--> sum(11::21::31::Nil)
--> 11 + sum(21::31::Nil)
--> 11 + (21 + sum(31::Nil))
--> 11 + (21 + (31 + sum(Nil)))
--> 11 + (21 + (31 + 0))
--> 11 + (21 + 31)
--> 11 + 52
--> 63 = (11 + (21 + (31 + 0)))
```

Exercise: Sum the Elements of a List

With a different zero element

```
1 def sum (xs:List[Int], z:Int = 0) : Int =  xs match
2   case Nil      => z
3   case x::rest => x + sum (rest, z)
```

```
1 val xs = List(11,21,31)
2 sum (xs)
```

```
sum(11::21::31::Nil)
--> sum(11::21::31::Nil, 0)
--> 11 + sum(21::31::Nil, 0)
--> 11 + (21 + sum(31::Nil, 0))
--> 11 + (21 + (31 + sum(Nil, 0)))
--> 11 + (21 + (31 + 0))
--> 11 + (21 + 31)
--> 11 + 52
--> 63 = (11 + (21 + (31 + 0)))
```

Exercise: Sum the Elements of a List

❓ Sum of elements in a list computing forward

```
1 def sum (xs:List[Int], z:Int = 0) : Int =  xs match
2   case Nil      => z
3   case x::rest => sum (rest, z + x)
```

```
1 val xs = List(11,21,31)
2 sum (xs)
```

```
sum(11::21::31::Nil)
--> sum(11::21::31::Nil, 0)
--> sum(21::31::Nil, 11)
--> sum(31::Nil, 32)
--> sum(Nil, 63)
-->
-->
-->
--> 63 = (((0 + 11) + 21) + 31)
```

Folds

generalize the `+` operation

```
1 def sum       (xs:List[Int], z:Int)                     : Int =
2   xs match
3     case Nil      => z
4     case x::rest => sum       (rest, z + x)
```

```
1 val xs = List(11,21,31)
2 sum        (xs, 0)
```

```
1 res1: Int = 63
```

Folds

💡 generalize the `+` operation

```
1 def foldLeft  (xs:List[Int], z:Int, f:((Int,Int)=>Int)) : Int =
2   xs match
3     case Nil      => z
4     case x::rest => foldLeft  (rest, f(z,x), f)
```

```
1 val xs = List(11,21,31)
2 foldLeft  (xs, 0, _+_) //  // (z:Int,x:Int) => z + x
```

```
1 res1: Int = 63
```

Folds

❓ Different element and aggregate type

```
1 def foldLeft  (xs:List[Int], z:String, f:(String,Int)=>String) : String =
2   xs match
3     case Nil      => z
4     case x::rest => foldLeft  (rest, f(z, x), f)
```

```
1 val xs = List(11,21,31)
2 foldLeft  (xs, "!", _ + " " + _) // (z:String,x:Int) => z + " " + x
```

```
1 res1: String = "! 11 21 31 "
```

Folds

Abstracting the types

```
1 def foldLeft  [Z,X] (xs:List[X], z:Z, f:((Z,X)=>Z)) : Z =
2   xs match
3     case Nil      => z
4     case x::rest => foldLeft  (rest, f(z,x), f)
```

```
1 val xs = List(11,21,31)
2 foldLeft  (xs, "!", _ + " " + _) // (z:String,x:Int) => z + " " + x
```

```
1 res1: String = "! 11 21 31"
```

Fold Left vs. Fold Right

Fold Left

```
1 def foldLeft  [Z,X] (xs:List[X], z:Z, f:((Z,X)=>Z)) : Z =
2   xs match
3     case Nil      => z
4     case x::rest => foldLeft  (rest, f(z,x), f)
```

```
1 val xs = List(11,21,31)
2 foldLeft  (xs, "!", _ + " " + _) // (z:String,x:Int) => z + " " + x
```

```
1 res1: String = "! 11 21 31"
```

Fold Right

```
1 def foldRight [X,Z] (xs:List[X], z:Z, f:((X,Z)=>Z)) : Z =
2   xs match
3     case Nil      => z
4     case x::rest => f (x, foldRight (rest, z, f))
```

```
1 val xs = List(11,21,31)
2 foldRight (xs, "!", _ + " " _) // (x:Int,z:String) => x + " " + z
```

```
1 res1: String = "11 21 31 !"
```

Folds Builtin in Lists

- Scala `List` class has `fold` methods (curried!)

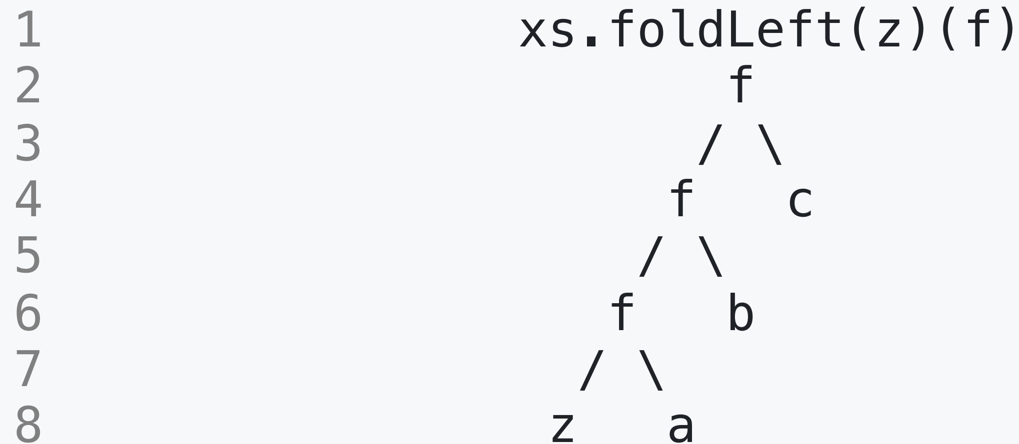
```
1 xss.foldLeft (0) ((z,xs) => z + xs.length)
```

Fold Left vs. Fold Right

```scala
def foldLeft [Z,X] (xs:List[X], z:Z, f:((Z,X)=>Z)) : Z =  xs match
  case Nil      => z
  case x::rest => foldLeft (rest, f(z,x), f)
```

```scala
def foldRight [X,Z] (xs:List[X], z:Z, f:((X,Z)=>Z)) : Z =  xs match
  case Nil      => z
  case x::rest => f (x, foldRight (rest, z, f))
```

```scala
val xs = List(a, b, c)
foldLeft (xs, z, f) === f( f( f(z,a),b),c)
foldRight(xs, z, f) === f(a, f(b, f(c,z)))
```



```
          xs.foldLeft(z)(f)       xs.foldRight(z)(f)
                 f                       f
                / \                     / \
               f   c                   a   f
              / \                         / \
             f   b                       b   f
            / \                             / \
           z   a                           c   z
```

Fold vs. Reduce

Fold

- Starts from a user-provide initial value
- Result type may differ from collection element type

```
1 List(1,2,3).fold(0)(_ + _)
```

- Change result type in a fold, provide initial element of result type

```
1 List(1,2,3).fold("!")(_.toString + _)
```

Reduce

- Starts from first element of collection
- Result type same as collection element type

```
1 List(1,2,3).reduce(_+_)
```

- Change result type with map before reduce, add initial element

```
1 "!" + List(1,2,3).map(_.toString).reduce(_+_)
```

Folds are Universal

```
1 def sum            (xs: List[Int])
2 def prod           (xs: List[Int])
3 def or             (xs: List[Boolean])
4 def and            (xs: List[Boolean])
5 def append   [X] (xs: List[X])(ys: List[X])
6 def flatten [X] (xs: List[List[X]])
7 def length   [X] (xs: List[X])
8 def reverse [X] (xs: List[X])
9 def map    [X,Y] (xs: List[X], f: X=>Y)
10 def filter  [X] (xs: List[X], f: X=>Boolean)
11 def reduce  [X] (xs: List[X], f: (X,X)=>X)
```

Summary

- Folds are universal functions to combine list elements into an aggregate result

Fold from the left **`foldLeft`**

- Zero element combined with head

```
xs.foldLeft(z)(f)
         f
        / \
       f   c
      / \
     f   b
    / \
   z   a
```

Fold from the right **`foldRight`**

- Zero element combined with last

```
xs.foldRight(z)(f)
         f
        / \
       a   f
          / \
         b   f
            / \
           c   z
```

Lots of examples, tutorial on [universality of folds]

Exceptions when Folding

- Example: turn list of tuples into sum of divisions

```
1 List(4->2,9->3).foldLeft(0) { case (acc,(n,d)) => acc + n/d }
```

- Division by 0

```
1 List(4->2,5->0,9->3).foldLeft(0) { case (acc,(n,d)) => acc + n/d }
```

- Collect first exception in `Either`

```
1 List(4->2,5->0,9->3).foldLeft[Either[Throwable,Int]](Right(0)) {
2     case (acc,(n,d)) => acc.flatMap {
3         a => try
4               Right(a + n/d)
5             catch
6               e => Left(e)
7     }
8 }
```

Folds are Universal

```
 1 def sum          (xs: List[Int])                 = xs.foldLeft(0)(_+_)
 2 def prod         (xs: List[Int])                 = xs.foldLeft(1)(_*_)
 3 def or           (xs: List[Boolean])             = xs.foldLeft(false)(_||_)
 4 def and          (xs: List[Boolean])             = xs.foldLeft(true)(_&&_)
 5 def append  [X] (xs: List[X])(ys: List[X])   = xs.foldRight(ys)(_::_)
 6 def flatten [X] (xs: List[List[X]])          = xs.foldLeft(Nil:List[X])(_:::_)
 7 def length  [X] (xs: List[X])                = xs.foldLeft(0)((z,_)=>z+1)
 8 def reverse [X] (xs: List[X])                = xs.foldLeft(Nil:List[X])((zs,x)=>x::zs)
 9 def map   [X,Y] (xs: List[X], f: X=>Y)       = xs.foldRight(Nil:List[Y])(f(_)::_)
10 def filter  [X] (xs: List[X], f: X=>Boolean) = xs.foldRight(Nil:List[X])((x,zs)=>if f(x) then x::zs else zs)
11 def reduce  [X] (xs: List[X], f: (X,X)=>X) = xs.tail.foldLeft(xs.head)(f)
```