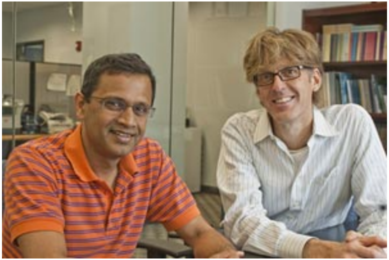
Eventual Consistency for CRDTs



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Chicago, USA

ESOP 2018

CRDTs?

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C = blah blah
R = mumble

DT = *Data Type*

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  - ✗`0`, ✗`1`: membership query returning false
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- Sequential interface:
  - ☺ `+0-0✗0`
  - ☹ `+0-0✓0`

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- Sequential interface:
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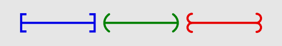
- ADT: contract between *implementor* and *client*
  - Implementor and client take turns

What about concurrent clients?

Linearizability (Herlihy/Wing 1990)

- "Each method call should appear to take effect instantaneously at some moment between its invocation and response." (Herlihy/Shavit 2008)

Specification 

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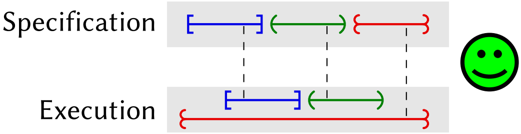

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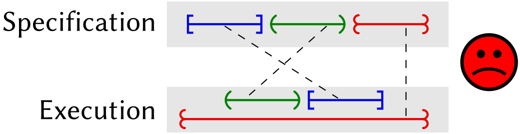

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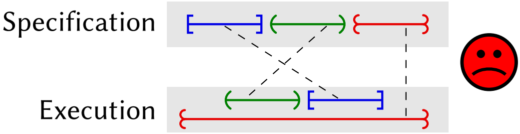

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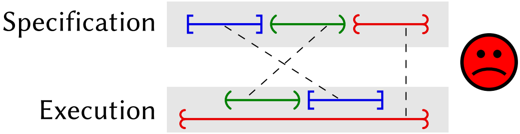

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- Implementor loses!
  - Intrinsically inefficient (Dwork/Herlihy/Waarts 1997)
    See also: CAP theorem (Gilbert/Lynch 2002)

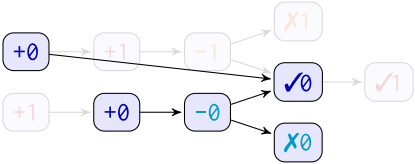
High performance?

High performance? ☹

R = Replicated

Replicated Sets: Add-Wins Set

- Specification of query:
  - ✗0 if every +0 followed by −0
  - ✓0 if some +0 not followed by −0
- Example Execution:



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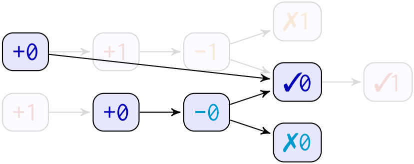

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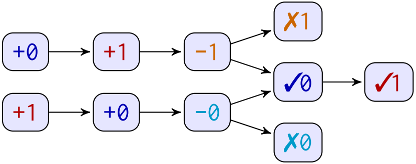

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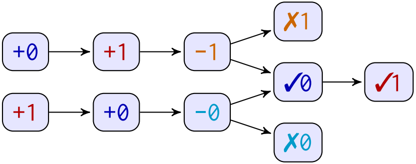

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Replicas that see same updates give same answers

Replicated Sets: Amazon Dynamo (?)

- Specification:

  …sloppy quorum…vector clock…

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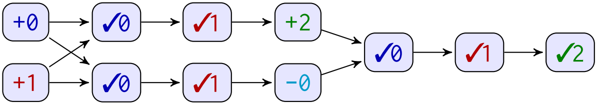

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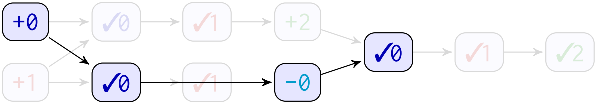

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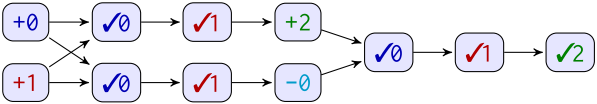

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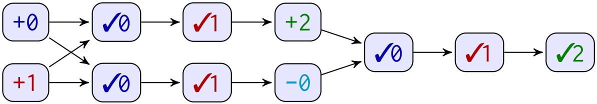

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- ☹ But SEC does not explain this

State Of Play

- Correctness Criterion: Strong Eventual Consistency (SEC)
  - ☺ Add-Wins Set Example
  - ☺ Amazon Dynamo Example
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Idea	Sequential	Replicated
Safety Termination		

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- This paper: What is a good notion of safety?

Safety?

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- This paper:

An extensional notion of safety for CRDTs
appealing only to the *sequential* spec

This talk: From Linearizability to CRDTs in 5 relaxations

Relaxations:

- Real time: Distributed system
- Order after an accessor: Update serializability
- Order between independent updates: Preserved Program Order
- Linearize labels, not events: Punning
- Quotient specification by observational equivalence: Stuttering

This talk: From Linearizability to CRDTs in 5 relaxations

Evidence that definition is the “right” one (in paper)

- ☺ Simulation-based characterization
  - ☺ *Most General CRDT*, expressed as Labelled Transition System
  - ☺ Compositionality and Substitutivity results
  - ☺ Validation of CRDT Graph built using CRDT sets
- ☺ Corner cases
  - ☺ Updates to one replica only $\Rightarrow$ linearizable
  - ☺ Permutation equivalence in spec $\Rightarrow \ldots$
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  - ☺ Add-Wins Set (Shapiro/Preguiça/Baquero/Zawirski 2011)
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  - Collaborative Text-Editing Protocol (Attiya/Burckhardt/Gotsman/Morrison/Yang/Zawirski)
- Validates *every possible* CRDTs
  - Def of CRDT does not mention sequential spec
  - Our def = proposal for meaning of CRDT

Components of Safety

- *Linearization*: response must be consistent with some spec string
  - List replica that sees `put(0)` and `put(2)` may respond
    - ☺ `q=[0,2]`
    - ☺ `q=[2,0]`
    - ☹ `q=[]`
    - ☹ `q=[1,0,2]`
- *Monotonicity*: responses evolve sensibly
  - List replica in state `q=[0,2]`, may evolve to
    - ☺ `q=[0,1,2]`, due to arrival of `put(1)`
    - ☹ `q=[2,0]`, no support for delete or reorder

Relaxation 1: Order in Distributed Systems

- v is *valid for* Σ if …

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  Order of LPO = non-overlapping method calls (real time)

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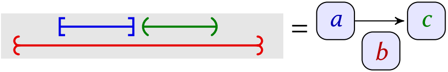

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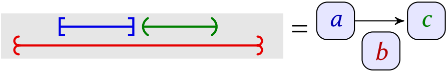

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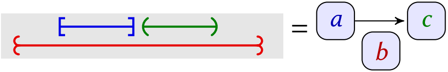

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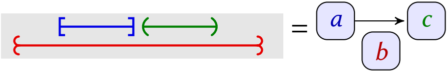

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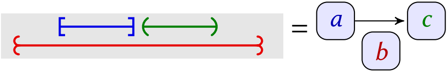

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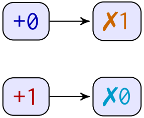
$C(v)$ = downclosed sets



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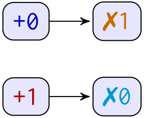

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$C(v)$ = *pointed* downclosed sets

$$\boxed{\texttt{+0}} \longrightarrow \boxed{\texttt{✗1}}$$

$$\boxed{\texttt{+1}} \longrightarrow \boxed{\texttt{✗0}}$$

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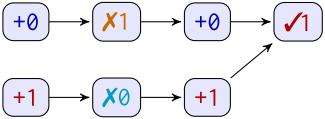

- Cannot linearize ✗0 and ✗1 together
- When linearizing ✓1, must not include both ✗0 and ✗1

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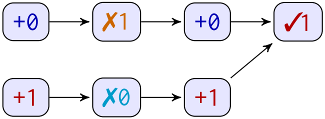
$C(v)$ = *pointed update*-downclosed sets



- Cannot linearize ✗0 and ✗1 together
- When linearizing ✓1, must not include both ✗0 and ✗1

Relaxation 2: Update Serializability

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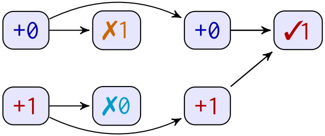

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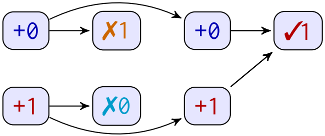

- Cannot linearize ✗0 and ✗1 together
- When linearizing ✓1, must not include both ✗0 and ✗1
- State of prior art (Burckhardt/Leijen/Fähndrich/Sagiv 2012)
  Cf. Update serializability: global order for updates
  (Hansdah/Patnaik 1986, Garcia-Molina and Wiederhold 1982)

Relaxation 3: Preserved Program Order

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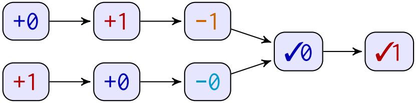

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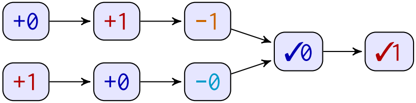

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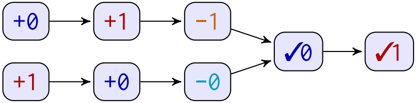

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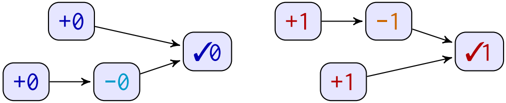

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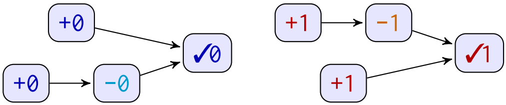

- Cannot linearize -1 and -0 together
- Cf. Preserved Program Order in relaxed memory models

(Higham/Kawash 2000, Alglave 2012).

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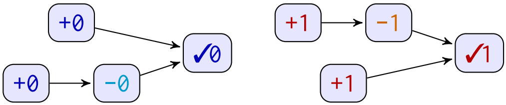

- Cannot linearize -1 and -0 together
- Cf. Preserved Program Order in relaxed memory models (Higham/Kawash 2000, Alglave 2012).
- Independency is a property of the *specification*

Relaxation 4: Puns

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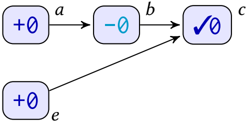

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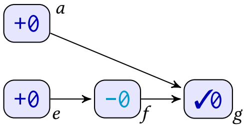

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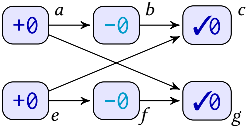

- Linearization must have $\texttt{-0}^b$ before $\texttt{+0}^e$
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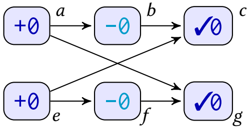

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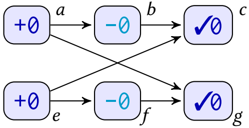

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$+0^a$, $+0^e$	:	$+0$
-0^b, -0^f	:	$+0-0$
$✓0^c$, $✓0^g$	:	$+0-0+0✓0$

Relaxation 4: A Bad Joke

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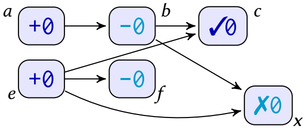

- Update order +0-0+0-0 with subsequences:

✓0^c	:	+0-0+0✓0	(✗0^c requires -0 *between* the +0s)
✗0^x	:	+0+0-0✗0	(✗0^x requires -0 *after* the +0s)

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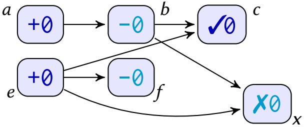

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✓0^c	:	`+0-0+0✓0`	(✗0^c requires `-0` *between* the `+0`s)
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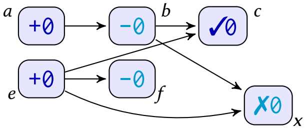

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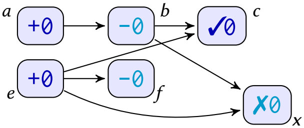

- Update order +0-0+0-0 with subsequences:

✓0[c]	:	+0-0+0✓0	(✗0[c] requires -0 *between* the +0s)
✗0[x]	:	+0+0-0✗0	(✗0[x] requires -0 *after* the +0s)

- Execution disallowed by monotonicity
  {+0[a], -0[b], +0[e]} cannot be linearized to satisfy both ✓0[c] and ✗0[x]

Relaxation 5: Observationally Equivalent Specifications

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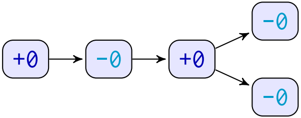

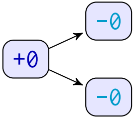

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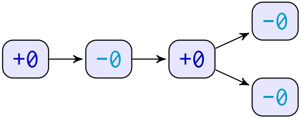

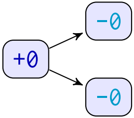

- Should this linearize to `+0-0-0+0-0+0-0-0`, or
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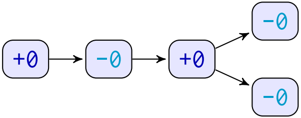

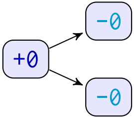

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- These are *observationally equivalent*
  Cf. stuttering equivalence (Brookes 96)

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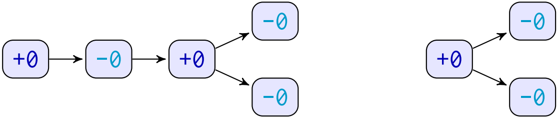

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- These are *observationally equivalent*
  Cf. stuttering equivalence (Brookes 96)
- Observational subsequence is a property of the *specification*

Safety: Summary

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$\mathcal{C}(v) =$ pointed dependent-update-downclosed sets, for accessors
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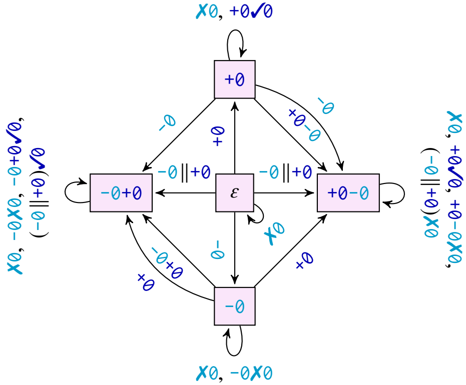
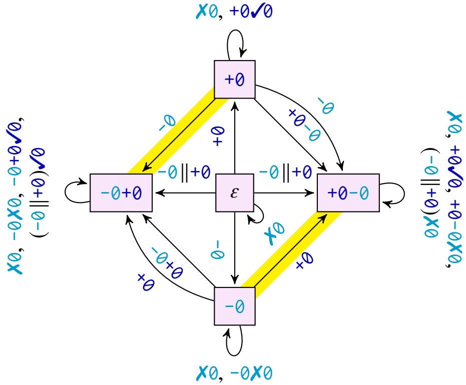
- Relaxations from linearizability:
  - Real time: Distributed system
  - Order after an accessor: Update serializability
  - Order between independent updates: Preserved Program Order
  - Linearize labels, not events: Punning
  - Quotient specification by observational equivalence: Stuttering

The Most General CRDT



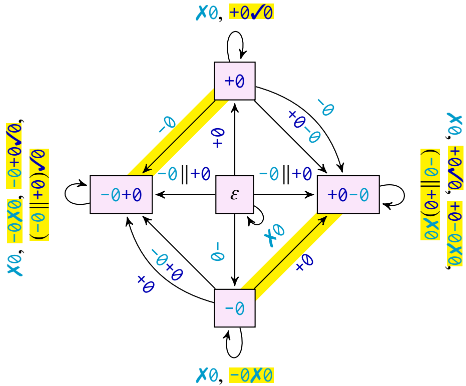
- What is the programmer model?
  Interacting with any CRDT implementation, for any specification
- Example for Set, with single +∅ and −∅
  LTS with labels = LPOs showing client history
  Maximal elements = new client actions

The Most General CRDT



- Contrast with linearizability
  - Updates may come out of order

The Most General CRDT



- Contrast with linearizability
  - Updates may come out of order
  - Accessors don't cause change of state

This talk: Definition of *safe* execution for CRDTs

In paper:

- ☺ Simulation-based characterization
  - ☺ *Most General CRDT*, expressed as Labelled Transition System
  - ☺ Compositionality and Substitutivity results
  - ☺ Validation of CRDT Graph built using CRDT sets
- ☺ Corner cases
  - ☺ Updates to one replica only $\Rightarrow$ linearizable
  - ☺ Permutation equivalence in spec $\Rightarrow \ldots$
- ☺ Validates all known CRDTs
  - ☺ Add-Wins Set (Shapiro/Preguiça/Baquero/Zawirski 2011)
  - ☺ Collaborative Text-Editing Protocol (Attiya/Burckhardt/Gotsman/Morrison/Yang/Zawirski)
- ☹ Validates *every possible* CRDTs
  - ☹ Def of CRDT does not mention sequential spec
  - ☺ Our def = proposal for meaning of CRDT